ESTAR INTERVENTION

Tier 2 Mathematics Intervention

Module: *Multiplication & Division of Whole Numbers (MDWN)*

Teacher Lesson Booklet

The Meadows Center for Preventing Educational Risk—Mathematics Institute
The University of Texas at Austin

The Meadows Center

FOR PREVENTING EDUCATIONAL RISK

THE UNIVERSITY OF TEXAS AT AUSTIN

COLLEGE OF EDUCATION

Mathematics Institute for Learning Disabilities and Difficulties

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25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 1

Reviewing Strategies for Fact Fluency

<table>
<tr><td>Lesson Objectives</td><td colspan="2">• The student will build fluency with multiplication facts using various strategies to solve.
• The student will recall steps from strategies in order to solve unknown multiplication problems.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">automatic, factors, product</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 1-15)
• Timer (optional)</td><td>• Student Booklet (pp. 1-8)
• Multiplication Connect 4 game board and cards (1 set per student pair)
• 10 transparent counters: 5 of 1 color and 5 of a different color (per pair of students)</td></tr>
</table>

Preview

Say: **Today we will review basic fact strategies to help us solve difficult facts that we have not mastered.**

Engage Prior/Informal Knowledge — Time: 3 min

Review the automaticity of facts. Have students identify the facts they are fluent in and the facts that still require thought and practice.

Have student turn to the *Engaged Practice Sheet.*

Say: **When you are automatic with a fact, you know it just by looking at it and you don't have to use your fingers or a strategy to find the answer.**

Fill in the blank multiplication table. You have 2 minutes to fill in all the facts you know automatically. Answer only the facts you know automatically.

Look over the students' table after the 2 minutes have passed. Be encouraging that they have made progress and remind students that it takes time to master all the multiplication facts.

Modeled Practice — Time: 8 min

1. Review multiplication and division strategies for solving unknown facts with factors of 6 or 7, as taught in Module: Multiplication and Division fact Strategies.

   Have students turn to ***Modeled Practice Sheets #1 and #2.*** Review the break-apart strategy by solving 6×7 in 2 different ways. First, solve 6×7 by breaking apart 6 into 1 and 5. Then, solve it again by breaking apart 7 into 2 and 5. The teacher and students will complete the steps together as the lesson progresses.

Say: **When we come across a multiplication fact we don't know quickly, or automatically, we can use a strategy to solve.**

6 × 7 can be a challenging fact, and strategies can be used to solve. We will work out this problem twice to review 2 different strategies to solve.

When we see a 6 or a 7 as a factor in a multiplication problem we can use the break-apart strategy. What strategy can we use? *(break apart)*

What is the first step of the break-apart strategy? *(break apart a factor)* **First, we can break 6 into 2 factors we know. What are 2 parts that we can break 6 into to make new facts that we know?** *(1 and 5)*

Teacher Note
For students who need concrete representation, use tiles to create an array. Have the students physically break (or separate) the array into 2 new arrays to visually see how the strategy works.

Say: **What is the second step in the break-apart strategy?** *(multiply by the other factor)*

What is the other factor? *(7)* **What is 1 × 7?** *(7)* **Write it.**

What is 5 × 7? *(35)* **Write it.**

What is the third step in the break-apart strategy? *(add the products)*

What is 7 + 35? *(42)* **What is 6 × 7?** *(42)* **Write it.**

Can we also solve 6 × 7 by breaking apart 7? *(yes)* **Why?** *(accept reasonable answers; we can break apart either factor, but have to complete all other steps)* **Let's try to solve 6 × 7 by breaking apart the 7.**

Have students turn to ***Modeled Practice Sheet #2***. Work 6 × 7 by breaking apart the 7. Fill in the blanks as you work through the problem with students.

Say: **What is step #1 in the break-apart strategy?** *(break 1 factor apart)*

What 2 parts can we break 7 into to make 2 factors to multiply quickly or automatically? *(2 and 5)* **Write it.**

What is step #2? *(multiply by the other factor)* **What are the multiplication problems?** *(6 × 2 and 6 × 5)* **Write it.**

What is step #3? *(add the products)*

What are the 2 products? *(12 and 30)* **Write it.**

What is 12 + 30? *(42)* **What is 6 × 7?** *(42)* **Write it.**

Turn to a math partner and explain which way you think works best for you when solving 6 × 7. *(allow students to explain why they prefer 1 method over the other)*

2. Review strategies for solving unknown facts with factors of 4 or 9 taught in the previous module.

Have students turn to ***Modeled Practice Display #3 and #4***. Review the "make 10 subtract the factor" strategy and the doubling strategy by solving 9 × 4 in 2 different ways. First, solve 9 × 4 by making 10 and subtracting the factor. Then, solve it again using the doubling strategy for 4s. The teacher and students will complete the steps together as the lesson progresses.

Say: **Here is another fact that may not be automatic, so we can use a strategy to solve. What are the 2 factors in this problem?** *(9 and 4)*

What is 1 strategy we can use to solve a fact with 9 as a factor? *(make 10 subtract the factor)* **First, let's solve using the "make 10 subtract the factor" strategy.**

We know that 9 = 10 – 1.

What is the first step in the "make 10 subtract the factor" strategy? *(think of 9 as 10)*

What is the second step? *(multiply the other factor by times 10)*

What is the other factor? *(4)* **What is 4 × 10?** *(40)* **Write it.**

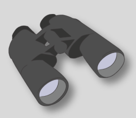
What is the last step in this strategy? *(subtract the other factor)*

What is 40 – 4? *(36)* **What is 9 × 4?** *(36)* **Write it.**

Why did we subtract instead of add when using the "make 10 subtract the factor" strategy? *(because we think of 9 as 10, 9 is 1 set less than 10, not 1 set more)*

Using *Modeled Practice Sheet #4*, work 9 × 4 using the doubling strategy for 4s. Fill in the blanks as you work through the problem.

Watch For



Students may have difficultly visualizing the 9 × 4 array in their heads. For students who cannot picture the array and therefore cannot see doubling 9 and doubling it again, draw an array on the whiteboard. Then, circle the 2 identical halves of the array for students to see 2 × 9 and 2 × 9.

Say: **Let's solve the same problem using the doubling strategy for 4s.**

What is the first step in the doubling strategy? *(think of 4 as 2 × 2)* **Write it.**

What is the second step? *(double the factor)* **What is the other factor we need to double?** *(9)*

What is 9 doubled? *(18)* **Write it**.

How many groups of 2 is 4? *(2)* **We will need to double the number again.**

We doubled 9 once to get 18. What is it doubled again, 18 + 18? *(36)* **Write it.**

Watch For	**Students may not be able to double 18 in their heads. If not use this think-aloud strategy: to double 18, I am going to think of 18 as 10 plus 8. Then I ask myself, what is 10 + 10? 20. What is 8 + 8? 16. What is 20 + 16? 36.**

Say: **Explain to your math partner which way you think works best for you when solving 9 × 4.** *(allow students to explain why they prefer 1 method over the other)*

Practice — Time: 8 min

Activity 1: Multiplication Connect 4 – students will use the *Multiplication Connect 4 game board and Multiplication Connect 4 game cards*, counters, and a whiteboard with marker to practice multiplication facts.

The goal of the game is to claim 4 squares in a row (horizontally, vertically, or diagonally) by using factor cards to create equations whose product appears on the game board. Students may need to use whiteboards to work out the steps to solve. Students should play in pairs and use 1of their sheets for each round they play.

Place the cards in a pile face down. Player 1 picks 3 cards from the pile. From the 3 cards, the student chooses only 2 to create a multiplication problem. The student must lay down cards and state the equation with the product. Then the student places a counter on the game board to cover the product they just found and place the cards in a discard pile.

Player 2 will repeat this same process. Once a product is covered it cannot be covered again. Once the cards have all been drawn, a student may shuffle the cards and place them facedown to continue the game.

During the game ask such question as:

- Which strategy did you use to determine your answer?
- Knowing 1 of your numbers is 5, how can you figure out what the other factor is to get a product of 30?

- What if I needed 12 to win? Which cards would I need to make 12? *(2 × 6 or 3 × 4)*
- Which 2 of your factors could be multiplied to claim a winning square?
- If your opponent has 3 counters on the board and it was your turn, what should you do?

Activity 2: Have students turn to the ***Practice Sheet*** on page 6. Students will complete the problems with their partner.

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Say: **Work with your math partner to complete the problems on your sheet.**

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes solving multiplication problems. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 2

Multiplying by Powers of 10

Lesson Objectives	• The student will multiply whole numbers by 10 and by powers of 10. • The student will develop and utilize mathematic language involved with multiplying by 10 and by powers of 10.	
Vocabulary	**multiple:** the products of the number and other factors (for example, the multiples of 2 are 2, 4, 6, 8, 10, 12, etc.) **powers of 10:** multiplying whole numbers by multiples of 10 (e.g., 10, 100, 1,000) **whole number:** any number used for counting that is not a fraction or has a decimal	
Reviewed Vocabulary	factor, product	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 16-21) • Whiteboard with green and red markers • Base-10 materials: 4 thousands, 4 hundreds, 4 tens	• Student Booklet (pp. 9-11) • Multiplication table (1 per student) • Colored marker or highlighter (1 per student)

Preview

Say: **Today we will multiply by 10 and the *powers of 10*. The *powers of 10* are numbers such as 100 and 1,000.**

What is another number that is a *power of 10*? *(10,000, 100,000, 1,000,000)*

Engage Prior/Informal Knowledge **Time: 3 min**

For review, have students practice reading, writing, and adding large numbers into the thousands period.

Write the following numbers on the whiteboard or use pre-made materials. Have students read the numbers.

- 5,000
- 30,000
- 200,000

Have students write numbers in the thousands. Read the following numbers and have the students write them on their whiteboards.

- 14,000
- 1,700
- 29,000
- 500,000

Have students add numbers in the thousands. Write the following on the whiteboard. Ask students to find the sum using mental math.

- 3,000 + 4,000
- 10,000 + 70,000
- 300,000 + 300,000

Modeled Practice **Time: 8 min**

1. Use base-10 materials to show and discuss multiplying by 10 and the powers of 10.

Distribute a *Multiplication Table* to each student.

Note to Teacher
In the discussion, help students see that multiplying by powers of 10 is very predictable and useful because our number system is based on 10.

Say: **Let's look at the *multiples* of 10. *Multiples* of a number are the products of the number and other factors. Thinking about 2 × 1, what is 2 × 1?** *(2)* **What is 2 × 2?** *(4)* **Both 2 and 4 would be *multiples* of 2. What is another *multiple* of 2?** *(answers will vary; 6; 8)*

A *multiple* is different than simply skip counting. We can skip count by tens starting at any number, like 22, 32, 42, but *multiples* have 1 factor that is the same. What is 10 × 1? *(10)* **What is 10 × 2?** *(20)* **These are *multiples* of 10.**

Look at the multiplication table. Find the column with 10. Read the *multiples* of 10.

What do these *multiples* of 10 have in common? *(the digit in the ones place is a zero)*

Place 4 tens on the table. Write the addition and multiplication sentence on the whiteboard as students answer the questions.

Say: **I have 4 groups of 10.**

What repeated addition problem does this model represent? *(10 + 10 + 10 +10)*

What multiplication problem does this model represent? *(4 × 10)*

Count by 10s. What is the total? *(10, 20, 30, 40)*

What is the total for 4 groups of 10? *(40)*

On the whiteboard, write "4" and then "40" underneath it, lining up the place values.

Say: **A *whole number* is any number used for counting that does not have a fraction or a decimal. 4 and 40 are *whole numbers*. What are other examples of whole numbers?** *(answers will vary; any number that does not include a fraction or decimal)*

When multiplying by 10, the *whole number* shifts one place value to the left, or to the tens place. The *whole number* shifts because the value of that number is now 10 times the value of the ones place. We write a "0" to show that 4 is now 10 times greater, or 40.

How did the value of the *whole number*, 4, change when multiplied by 10? *(the whole number value changed, not the whole number; its value is now 40, rather than just 4)*

2. Discuss multiplying by 100.

Replace the 4 tens with 4 hundreds.

Say: **What is the value of each hundred?** *(100)*

How many groups of 100 do I have? *(4)*

What multiplication problem does this model represent? *(4 × 100)*

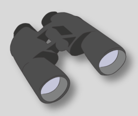
What is the total? *(400)* **How do you know?** *(counted by 100; accept other reasonable answers)*

The *whole number* shifted to the left 2 places, because 4 is now 100 times the value. To show the value change, we must now include 2 zeros in the number—a 0 in the tens place and a 0 in the ones place. How did the value of the *whole number*, 4, change when multiplied by 100? *(the value is now 100 times greater)*

When multiplying by 100, the *whole number* shifts 2 places to the left, to the hundreds place.

On the whiteboard under the 40, write "400," lining up the place values.

Watch For



If a student replies that when multiplying by a multiple of 10, the answer will <u>always</u> have a certain number of 0s, warn the student to be careful when using the word "always." Tell students to hold onto the thought to see if it is <u>always</u> true or not.

3. Discuss multiplying by 1,000.

Replace the 4 hundreds with 4 thousands.

Say: **What is the value of each thousand?** *(1,000)*

Count the hundreds in the thousand. How many groups of 100 are in 1,000? *(10)*

How many groups of 10 are in 1,000? *(100)*

Teacher Note

If students struggle with this question, write out the multiplication sentences "10 × 100 = 1,000" and "? × 10 = 1,000." Remind students of the Commutative Property of Multiplication to help them make the connection between the 2 facts.

Say: **What multiplication problem does 4 thousands represent?** *(4 x 1,000)*

Make a prediction for the total. *(ask a few students to share their predictions)*

Count by 1,000s to find the total. Ready, count: "1,000, 2,000, 3,000, 4,000."

Was your prediction correct?

How did the value of the *whole number*, 4, change? *(it is now 1,000 times greater)*

When multiplying by 1,000, the *whole number* shifts 3 places to the left because 4 is now 1,000 times the value.

How many digits are in a number in the thousands? *(4)* **What digits are in the number 4,000?** *(4, 0, 0, 0)*

On the whiteboard under 400, write "4,000," lining up the place values.

4. Use the colored markers to help identify the pattern when multiplying by powers of 10.

   On the whiteboard, write "10 × 10," writing the first 10 in black and the second 10 in red (or color of your choice).

Say: **What is 10 × 10?** *(100)*

Write "= 100" in black.

Say: **When we multiply by 10, how much does the value of the number increase?** *(10 times)* **When the value of a number increases 10 times, the *whole number* shifts 1 place to the left.**

Trace over the last "0" in 100 in red or erase and rewrite it in red.

Say: **Did the *whole number* 10 shift 1 place to the place?** *(yes)*

Is the same *whole number* 10 still written? *(yes)*

Does the product end in a 0? *(yes)*

Why do we only write 1 zero in the ones place? *(multiplying by 10)*

What is the pattern for multiplying by 10 and the *powers of 10*? *(the value of the whole number increases the number of tens being* *multiplied; the whole number shifts left and then is followed by zeros)*

Practice **Time: 8 min**

Activity 1: Students will practice multiplying by 10 and powers of 10. Students will use the marker or highlighter to show their work. Work along with the students at first, then fade teacher assistance.

Have students turn to the ***Practice Sheet*** on page 9 and complete the first 2 problems. Distribute a colored marker or highlighter to students.

Say: **On your sheet, as you solve the multiplication problems use the marker or highlighter to show your work. Use your pencil to show the *whole number*, then the marker or highlighter to show how the *whole number* changed after it was multiplied by a *power of 10*.**

Check that students have only written the "60" in pencil on problem 3. Demonstrate on your own whiteboard.

Ask such question as:

- What do you know about multiplying a *whole number* by 100? *(the whole number shifts to the left to show that the value is now 100 times greater)*
- What is 100 × 7? *(700)*
- To write the answer "700," what color do you use to write the whole number, "7"? *(pencil)*
- What do you use to write the 0s? *(marker or highlighter)*
- What happened to the *whole number* 7 when it was multiplied by 100? *(it shifted to the left to show that the value increased 100 times)*

Activity 2: Have students continue to work with their partner answering the rest of the problems from the ***Practice Sheet***.

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Say: **Working with your partner, practice multiplying by *powers of 10*. Identify the shift to show how the value of the *whole number* changes.**

Use the marker or highlighter to answer the problems.

Allow students time to complete their work. Use mathematical language from the lesson to check students' understanding.

Say: **Explain the pattern that 60 × 1,000 follows.** *(accept answers that include the whole number 60 shifting to show that the value of 60 is now 1000 times greater)*

How is this different than when we multiply 60 × 100? *(have students compare it to the number of places 60 moved when multiplied by 100)*

Activity 3 (Optional): Provide additional guided practice to students who are unable to mentally multiply by 100 or 1,000. Use a non-example for students who are struggling. For example, write on the board "40 × 100 = 400."

Say: **100 × 40 = 400. Is this true or false?** *(false)*

Does this answer make sense? Why not? What is my mistake? *(accept answers, such as: the number 40 only moved 1 place value to the left instead of 2; there needs to be 2 0s behind the 40 instead of 1; 40 already has a 0, but you still have to add 2 more)*

Is 40 now 100 times greater? *(no)* **How do you know?** *(the value of a 3-digit number is 100, the value of 4 groups of 10 is now 100 times greater)*

What number should I have multiplied to 100 to get 400? *(4)*

Watch For

If students struggle with shifting the whole number left and understanding the number of 0s that will be in the product, consider teaching the lesson using place-value charts. Students should write or build the original number in the chart, add the multiple of 10 to show the value increasing, and then write the new number or the product.

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: The students share their answer with the group. The teacher provides corrective feedback using mathematical language from the lesson. The students mark total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 3

Multiplying Multiples of 10 and Basic Facts

<table>
<tr><td rowspan="2">Lesson Objectives</td><td colspan="2">• The student will use basic facts and multiples of 10 to solve division and multiplication problems.
• The student will explain using mathematically correct terms the change a number makes when multiplied by 10 or a multiple of 10.</td></tr>
<tr></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">Associative Property of Multiplication, factor, multiple, powers of 10, product</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 22-31)
• Whiteboard with marker</td><td>• Student Booklet (pp. 12-16)
• Whiteboard with marker (1 per student)
• Place-Value Chart (optional)
• Expanded Fact cards (1 set per student pair)</td></tr>
</table>

Preview

Say: **Today we will extend and use our knowledge of multiplying with powers of 10 to multiply by multiples of 10. We will also relate multiplication to division to solve division equations with multiples of 10.**

Engage Prior/Informal Knowledge Time: 3 min

For review, have students mentally solve whole numbers times 10 or a power of 10.

Example problems:

100×13	4×1000	20×10	50×100

Ask questions such as:

- How does the value of 13 change when it's multiplied by 100? *(it increases 100 times its value)*
- By how many places does it increase? *(2 places, the product is 100 times the value of the start number)*
- When multiplying by 1,000, how many places does the whole number shift? *(3)*
- When a number is multiplied by 10, by how many places does the number increase? *(1, the product is 10 times larger)*
- When a number is multiplied by 1000, by how many places does the number increase? *(3, the product is a thousand times the value of the start number)*
- How did you solve 50 x 100? *(answers will vary; we know 50 would shift 2 place values to the left because when multiplying by 100, the whole number shifts 2 places; the product is 100 times larger)*

Modeled Practice Time: 8 min

1. Students will multiply a whole number times a multiple of 10. Start by reviewing the vocabulary term "multiple" from Lesson 2.

Say: **Multiples are the products of any 2 whole numbers. What are the multiples of 2?** *(2, 4, 6, 8…)*

What are some multiples of 10? *(10, 20, 30, 40, 50...)*

What do all multiples of 10 have in common? *(each ends with a 0)*

Is there an end to the list of multiples for a given number? *(no)*

Would 120 be a multiple of 10? *(yes)* **How do you know?** *(has a 0 in the ones place)*

Would 1,300 be a multiple of 10? *(yes)* **How do you know?** *(has a 0 in the ones place)*

2. Demonstrate multiplying multiples of 10 together.

Have students turn to the ***Modeled Practice Sheet***. The teacher and students should fill in the blanks as the lesson progresses.

Say: **What is 4 × 3?** *(12)* **Write "12" after 4 × 3.**

If 4 became 40, what changed? *(multiplied it by 10; the 4 shifted 1 place value to the left)*

Think of 40 as 4 × 10. Write "4 × 10" under 40.

If 3 became 30, what changed? *(multiplied it by 1; the 3 shifted 1 place value to the left)*

Think of 30 as 3 × 10. Write "3 × 10" under 30.

Think of 40 × 30 as 4 ×10 × 3 × 10. Do we have to multiply 4 × 10 first? *(no)* **Why?** *(can change the order of the factors in multiplication)*

We do not have to multiply the factors in this order. The associative property of multiplication states that we can change the grouping of factors without changing the product. We can move the factors around to create a simpler problem using facts we already know.

From the 4 factors in the problem, pair up the 2 factors you can multiply in your head. *(4 × 3 = 12 or 10 × 10 = 100)* **Write "4 × 3 × 10 × 10."**

Let's multiply 4 × 3 first because we already know the answer is 12. Write "12" under 4 × 3.

Next we can group the 10s together. What is 10 × 10? *(100)* **Write "100" under 10 × 10.**

What is the new multiplication problem? *(12 × 100)*

What is 12 × 100? *(1,200)*

What is 40 × 30? *(1,200)*

4 × 3 equals 12 and 40 × 30 equals 1,200 or 12 hundreds.

Explain how the 12 and 1,200 are similar, yet the values are different. *(accept reasonable answers; both numbers have 12, but the value is 100 times greater)*

3. Review multiplying factors whose product ends in 0.

Have students continue on the ***Modeled Practice Sheet***. The teacher and students should fill in the blanks as the lesson progresses.

Say: **What are the next 2 problems?** *(4 × 5 and 40 × 50)*

What is 4 × 5? *(20)* **Write it.**

To solve 40 × 50, how do we think of 40 and 50? *(40 as 4 × 10 and 50 as 5 × 10)* **Write it.**

We will follow the same steps as the last problem: first, multiply 4 × 5, then multiply 10 × 10. Write "4 × 5 × 10 × 10" under the original problem.

What is 4 × 5? *(20)* **What is 10 × 10?** *(100)* **Write it.**

Does 20 × 100 equal 200 or 2,000? *(2,000)*

When we multiply a number by 100, the whole number shifts 2 places to the left.

How many places did the 20 shift to make 200? *(1 place)*

20 times what number equals 200? *(10)* **We are trying to solve 20 times 100.**

How many places did the 20 shift to make 2,000? *(2 places)*

If 20 × 100 equals 2,000, what does 40 × 50 equal? *(2,000)*

Watch For	**Students may struggle to see the shift in place value. For example, the student cannot tell you how many places the whole number has shifted when multiplied by 10 or 100. To help students see the shift, write the numbers on a place-value chart: write "20," then on the line below write "200," and on the line below 200, write "2,000." This will provide the visual aid students may need to see the whole number shifting places.**

Practice Time: 8 min

Activity 1: Students will continue to practice multiplying by multiples of 10. Provide support and corrective feedback as students work on the *Practice Sheets* on pages 13 and 14.

Work the first problem together.

Say: **Read the first problem on your practice sheet together. Ready, read: "Mrs. Hern has 30 fourth grade math students. She ordered each student a pencil-top eraser, 2 folders, and 5 colored pens. Each eraser cost $0.20. How much did she spend on 30 erasers?"**

What is the question asking you to find? *(how much money Mrs. Hern spent on erasers)* **Underline it.**

What is the important information we need to solve this question? *(30 students, $.20 for each eraser)* **Circle it.**

We need to find the total. What operation should we use to find the total when we have equal groups? *(multiplication)* **What is the multiplication problem?** *(30 × 20)*

What basic fact, or 1-digit multiplication fact, can we use to help us solve? *(3 × 2 = 6)*

Explain how you solve 30 x 20. *(think of 30 as 3 x 10 and 20 as 2 x 10; multiply 3 x 2 = 6 and 10 x 10 = 100; multiply 6 x 100 = 600)*

What does 600 represent—600 dollars or 600 cents? *(600 cents)*

How many cents are in 1 dollar? *(100)*

How many dollars are 600 cents? *($6.00)*

Let's make sure we answered the question. How much did she spend on the 30 erasers? *($6.00)*

Continue to work on your practice sheet with your math partner.

Students will work in partners to complete the practice sheet. Ask students questions such as:

- Did you need to use a strategy to solve the fact?
- How many places did the whole number shift?
- How does solving the basic fact first help you in solving the expanded fact?

Activity 2: Use the ***Expanded Fact cards*** have students match the expanded facts cards with the facts cards. With a partner, students will place cards face up in a 3-by-4 array. The students are looking for a basic fact and the matching extended fact (for example, 2 × 6 matches with 20 × 60). If the cards match, the student must tell the group the product of each card in order to keep the cards. The second player then picks up 2 cards and continues the game. The player with the most cards wins. The student may write out the problem on a whiteboard or scratch paper if a strategy is needed to solve. The student should answer the basic fact first before attempting to answer the extended fact.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 4

Estimating Products of 2-Digit by 2-Digit Multiplication Problems

Lesson Objectives	• The student will use estimation to solve multiplication of 2-digit numbers times 2-digit whole numbers. • The student will use new vocabulary in descriptions and classroom communication.	
Vocabulary	**estimation:** an educated guess of the actual value **reasonable:** an answer that makes sense or is logical	
Reviewed Vocabulary	factors, product, rounding, whole numbers	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 32-37) • Calculator • Whiteboard with marker • Extended Fact cards	• Student Booklet (pp. 17-19) • Whiteboard with marker (1 per student) • Calculator (1 per pair of students)

Preview

Say: Today we will use rounding as a tool to *estimate* products.

Teacher Note
This lesson is not an introduction to rounding. The goal is to know when it may be appropriate to use rounding as a tool to estimate. If students do not know how to round, stop and reteach.

Engage Prior/Informal Knowledge Time: 3 min

Review multiplying by multiples of 10s using the *Extended Fact cards*.

Place the cards face down on the table. Each student will draw a card and then solve the problem either in their head or by using their whiteboard. Allow 1 minute for students to solve their individual multiplication problems. Then have students share their problem and the solution.

Ask such questions as:

- Which basic fact did you first answer to help you solve the extended fact?
- How did you break apart the factors?
- How did you rearrange the factors to create 2 simpler multiplication facts?
- How many places did the whole number shift to the left when you multiplied it by 100? *(2)*

Modeled Practice Time: 8 min

1. Review the importance of estimation.

Say: **It is important to *estimate,* or make a good, educated guess of the answer, before computing.**

Sometimes we make mistakes when multiplying numbers, but *estimating* will tell us if our answer is *reasonable. Reasonable* means that the answer makes sense or is logical. Think about 245 + 316. What would be a *reasonable estimate* of the answer, 550 or 250? (550) **Why?** *(answers will vary; 200 + 300 equals 500; 200 is less than the 2 numbers being added)*

We can use rounding to *estimate* products. When do you think we can use *estimation* in life? *(answers will vary; in the grocery store; getting party supplies)*

***Rounding* is used to *estimate* to a given place value. What is an example of *rounding*?** *(answers will vary)*

Fill in the blanks on the *Modeled Practice Sheet* as you work through the lesson. Have the students work along with you on their sheet.

Say: **We can use rounding as a tool to *estimate* the product of 37 × 28.**

On the top of the page we have the number line from 20 to 40. Where would 37 be on this number line? Use a hash mark to *estimate* where 37 would be on the number line.

Between which 2 multiples of 10 does 37 fall on the number line? *(between 30-40)*

Where would 28 be on this number line? Use a hash mark to *estimate* where 28 would be on the number line.

Check that students have marked 37 and 28 on the number line.

Say: **Between which 2 multiples of 10 does 28 fall on the number line?** *(between 20-30)*

Is 37 closer to 30 or 40? *(40)* **Write "40" underneath the arrow.**

Is 28 closer to 20 or 30? *(30)* **Write "30" underneath the arrow.**

Teacher Note

The number 1,200 can be read as "twelve hundred" as long as it is also read as "one thousand, two hundred."

Say: **Think about the last lesson and how we solved multiples of 10. Both 40 and 30 can be broken apart into some factor times 10. Which factors were multiplied by 10 to get 40 and 30?** *(4 and 3)*

How can we think of 40 × 30? *(4 × 10 × 3 × 10)* **Write it.**

First solve 4 × 3. What is 4 × 3? *(12)* **Write it.**

Next solve 10 × 10. What is 10 × 10? *(100)* **Write it.**

What is 12 × 100? *(1200)* **Write it.**

What is 40 × 30? *(1200)* **Write it.**

So, is the exact product of 37 × 28, 1,200? *(no)*

Provide a calculator to 1 student and ask the student to find the exact answer for 37 × 28. The student should get the answer 1,036.

What is 37 × 28? *(1,036)* **Is the exact product, 1,036, close to 1,200?** *(yes)*

Is 1,200 a *reasonable estimation*? *(yes)* **Why?** *(it is close to the exact answer)*

2. Solve another problem using rounding as an estimation tool.

Continue to fill in the blanks on the *Modeled Practice Sheet* as you work through the lesson.

Say: **Let's *estimate* to solve the problem 52 × 68. Instead of working out the actual answer, we will round the numbers to find an *estimated* answer quickly.**

Think about the number 52. Which 2 multiples of 10 is 52 between? *(50 and 60)*

Is 52 closer to 50 or 60? *(50)* **Why?** *(it is only 2 away from 50, but 8 away from 60)* **Write "50" on the line below the arrow.**

In the problem 52 × 68, we are going to *round* 52 to 50. How else we can think of 50? *(as 5 × 10)* **Write it.**

Think about 68. What 2 multiples of 10 is 68 in between? *(60 and 70)*

Is 68 closer to 60 or 70? *(70)* **Why?** *(it is only 2 away from 70, but 8 away from 60)* **Write "70" on the line below the arrow.**

How else can we think of 70? *(as 7 ×10)* **Write it.**

Read the problem now. *(5 × 10 × 7 ×10)* **How we can create 2 simpler problems in order to solve this problem quickly?** *(rearrange the factors to get 5 × 7 and 10 × 10)*

How can you solve 5 × 7 efficiently? *(skip count by 5s)* **What is 5 × 7?** *(35)* **Write it.**

What is 10 × 10? *(100)* **Write it.**

What is our *estimated* answer for 52 × 68? *(3500)* **Write it.**

Does 52 × 68 = 3500? *(no)*

Provide a calculator to 1 student and ask the student to find the exact answer for 52 × 68. The student should get the answer 3,536.

What is 52 × 68? *(3,536)* **Is the exact product, 3,536, close to 3,500?** *(yes)*

Is 3,500 a *reasonable estimation*? *(yes)* **Why?** *(it is close to the exact answer)*

***Estimation* can be used as a tool to check that your work is *reasonable*. We first *estimated* and then compared the exact product with the *estimation*. If our answer is different from our *estimation* by a large amount, then that can be a clue that we made a mistake in our calculation.**

With more practice, *estimation* will become automatic to you, allowing you to *estimate* mentally.

Practice — Time: 8 min

Activity 1: Students will practice estimation to find the product of a 2-digit by 2-digit fact. Have students turn to the *Practice Sheet* on page 18 and read the first problem together.

Say: **Let's read the first problem together. Ready? Read: "The whole school went on a trip to the aquarium. There were 17 buses, about 42 students, and 3 teachers on each bus. Estimate how many students went on the trip to the aquarium."**

What is the question asking us to find? *(the number of students that went on the trip)* **Underline it.**

What is the important information needed to solve the question? *(17 buses, 42 students)* **Circle it.**

Do we need to know that 3 teachers were on each bus? *(no)* **Why?** *(the question is asking for how many students)*

Do we need to find exactly how many students went on the trip? *(no)* **How do you know?** *(it says estimate)*

With your math partner, develop a plan for how to solve this problem.

Give students 1 minute to discuss how to solve the problem. Then bring the group back together to walk through the steps and the solution.

The students will work with a math partner to estimate the answer to the next 2 problems. After the estimated answer is found, the students may use a calculator to check that their estimation is reasonable.

Ask question such as:

- Which 2 multiples of 10 is 18 between? *(10 and 20)*
- Which multiple of 10 is 26 closer to? *(30)*
- When multiplying 20 times 30, which basic fact do you solve first? *($2 \times 3 = 6$)*

Monitor students' work and provide corrective feedback when necessary. Allow each group to explain how they solved the problem.

Activity 2: Students will work with a math partner practicing estimation.

Write "18," "26," "61," and "94" on the whiteboard. Students will pick numbers from the list that, when estimated, can produce the product 1,800.

Say: **Look at this list of factors. Using your skills of *rounding*, find 2 factors that have the *estimated* product of 1,800.**

Hint: there is more than 1 way to get close to 1,800. *($18 \times 94 \approx 18 \times 100 = 1{,}800$; $26 \times 61 \approx 30 \times 60 = 1{,}800$)*

Give students time to share their thinking and ways of estimating.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes *estimating* products. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

ESTAR INTERVENTION

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 5

Breaking Apart Arrays

Lesson Objectives	• The student will make arrays and break them apart to find partial products. • The student will correctly use mathematical language, including "factor" and "product" during discussions.	
Vocabulary	**partial-products method:** a method for multiplying multi-digit numbers by taking the base-10 decomposition of each factor and forming the products of all pairs of terms; the partial products are then added together	
Reviewed Vocabulary	array, distributive property, equation, factor, product	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 38-51) • Whiteboard with marker	• Student Booklet (pp. 20-26)

Preview

Say: **Today we will use our mathematical knowledge to break apart arrays to build understanding of multiplication of larger numbers.**

Engage Prior/Informal Knowledge — Time: 3 min

Students will practice needed skills for multiplying multi-digit numbers.

Have student solve addition problems in their heads by breaking the numbers into ten and ones.

Say: **We will practice thinking of numbers in expanded notation.**

How many tens are in 32? *(3 tens)*

How many ones are in 32? *(2 ones)*

What about the number 15, how many tens? *(1 ten)* **How many ones?** *(5)*

Let's add 32 + 15 mentally. 32 has 3 tens and 15 has 1 ten. How many tens altogether? *(4 tens)* **What is the value of 4 tens?** *(40)*

Think about the ones, 2 ones + 5 ones. How many ones altogether? *(7 ones)*

Now put it altogether. What is 40 + 7? *(47)* **What is 32 + 15?** *(47)*

Work through more problems as time permits, such as 44 + 23, 51 + 38, and 71 + 16.

Modeled Practice — Time: 8 min

1. Multiply 2-digit factors times 1-digit factors using partial products to build students' understanding of multi-digit multiplication.

   Use *Modeled Practice Sheet #1*, an array for 3 × 15 counters. The teacher and students should complete the steps together as the lesson progresses.

Say: **How many circles are in each column?** *(3)*

How many columns? *(15)*

What is the multiplication expression for the array shown? *(3 × 15)*

We need to find the total number of counters on the page, but counting each 1 will take time and many students may not know 3 × 15 automatically.

We can split one of the factors into 2 parts—tens and ones. Look at 15. How many groups of 10? *(1 ten)* **How many ones?** *(5 ones)*

Count over 10 columns to make 1 group of 10. Draw a line down to separate the 15 into 10 and 5, or 2 parts.

Watch For	Students may struggle to see the break in the larger array model. For example, students may become confused by the mention of 2 arrays. If this happens, have students circle each array or shade in 1 array to make it different from the other. This may help students see the 2 new arrays created from the old 1.

Say: **Now we have 2 smaller arrays. What is the multiplication expression for the first array?** *(3 × 10)* **What is the multiplication expression for the second array?** *(3 × 5)*

Draw a bracket under each array and write the multiplication equation.

What is 3 × 10? *(30)* **Write it.**

What is 3 ×5? *(15)* **Write it.**

Did you use a strategy to solve either of these facts? *(no; or skip counted)*

By breaking apart 1 factor into 2 known factors we were able to quickly or automatically multiply to find the total.

30 and 15 are called *partial products*. The word *partial* means incomplete, and the word *product*, we know, is the answer to a multiplication fact. What does *partial* mean? *(incomplete)*

To find these *partial products,* we broke the factor 15 into 2 parts—tens and ones—and multiplied each part by the other factor. How do we break apart the 2-digit number? *(into tens and ones)*

Where have we used these same steps before? *(break-apart strategy)*

In the break-apart strategy, we break 1 factor into 2 parts. We are using the same strategy, but now we break apart the 2-digit factor into tens and ones. When we use this strategy for 2-digit numbers we call it the *partial-products method*.

What is the last step in the break-apart strategy and also the *partial-products method*? *(add the products together to find the total)*

Write "30 + 15 =" on the sheet. 30 and 15 are the *partial products* because the work is not complete yet.

Complete the addition sentence after students have provided the answer. Do the same with the multiplication sentence.

Say: **What is 30 + 15?** *(45)* **Write it.**

What is 3 ×15? *(45)* **Write the multiplication equation under the addition equation.**

2. Practice the partial product method.

Use ***Modeled Practice Sheet #2,*** an array for 4 × 21. The teacher and students should complete the steps together as the lesson progresses.

Say: **What array is shown?** *(4 × 21)* **Write it.**

Do you know this fact automatically? *(no)*

What method or strategy could we use to solve this problem? *(partial-product method or break-apart strategy)*

How should we break apart the array to create partial products? *(into tens and ones)* **How many groups of 10 in 21?** *(2)* **How many ones?** *(1)* **Draw a line to split the array into 2 arrays: 4 by 20 and 4 by 1.**

Look at the 2 smaller arrays. What is the multiplication expression for each array? *(4 × 20 and 4 × 1)*

Bracket each array and write "4 × 20" and "4 × 1" under the appropriate array.

What is 4 × 20? *(80)* **Write it.**

What is 4 × 1? *(4)* **Write it.**

Remember, 80 and 4 are called *partial products*. What does partial mean? *(incomplete)*

We broke apart the factor 21 to create 2 arrays. How do we find the total amount of the original array? *(add the 2 partial products together)*

Write "80 + 4 =" under the array.

Complete the addition problem after students have provided the answer. Do the same for the multiplication problem.

Say: **What is 80 + 4?** *(84)* **Write it.**

What is 4 × 21? *(84)* **Write it.**

Think about the steps you just took. In your own words, describe how to solve multiplication problems using the *partial-products*/break-apart strategy.

Practice — Time: 8 min

Activity 1: Students will complete 1 problem without the use of an array and 1 problem with an array. Have student turn to the *Practice Sheet* on pages 22 and 23.

Say: **We will complete this first problem on your sheet together. First let's read the problem. Ready, read: "The grocery store has a peanut butter display. The display is organized in 6 rows with 15 jars of peanut butter on each row. How many total jars of peanut butters are on display?"**

What is the question asking you to find? *(the number of peanut butter jars on display)* **Underline it.**

What is the important information? *(6 rows, 15 jars)* **Circle it.**

If we wanted to draw an array, how many rows would it need? *(6)* **How many columns?** *(15)* **Do we need to draw an array, or can we still split the 2-digit factor into tens and ones?** *(answers will vary)*

What factor will we split? *(15)*

What is the value of the tens place in 15? *(10)*

What will you multiply 10 by to get the first *partial product*? *(6)*

What is this first multiplication expression? *(10 × 6)* **Write it.**

What is 10 × 6? *(60)* **Write it.**

What is the value of the ones place in 15? *(5)*

What is the multiplication expression to find the second *partial product*? *(5 × 6)* **Write it.**

What is 5 × 6? *(30)* **Write it.**

What do we do with the 2 *partial products*? *(add them together)*

What is the addition sentence? *(60 + 30)* **Write it.**

What is 60 + 30? *(90)* **Write it.**

What is 6 × 15? *(90)* **Write it.**

90 what? *(jars of peanut butter)*

Did you answer the question, how many jars of peanut butter are on display? *(yes, 90 jars of peanut butter)*

Complete the second problem with the array on your own. Break apart the array and find the 2 *partial products*.

Allow students time to complete the problem independently. Once everyone finishes review the students' work and their solutions.

Activity 2: Students will work with a math partner solving 2-digit by 1-digit multiplication using the partial-products method. In each pair of students, assign 1 student as "A" and 1 student as "B." Have students turn to the *Practice Sheets* on page 24.

Say: **With your math partner, you will take turns solving for the ones and solving for the tens. For the top problem on the page, student "A" will solve for the tens and student "B" will solve for the ones. Then switch for the bottom problem.**

Raise your hand if you will be solving for the tens.

Then work together to find the sum of the 2 *partial products*.

To check for understanding of the directions:

- Ask student "A" what will be the multiplication problem for the tens in the first problem. *(5 × 10)*

- Ask student "B" what will be the multiplication problem for the ones in the first problem. *(5 × 8)*

When partners have completed the work ask for solutions and explanations.

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 6

Using Area Models to Solve 2-Digit by 1-Digit Multiplication Problems

Lesson Objectives	• The student will use an area model to reinforce the partial-products method with 2-digit by 1-digit numbers. • The student will verbalize the connection between the area model and the partial-products method.	
Vocabulary	**dimensions:** the length and width of a shape **area:** the total number of square units inside of a shape	
Reviewed Vocabulary	estimation, distributive property, partial product	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 52-69) • Whiteboard with marker	• Student Booklet (pp. 27-35) • 2-Digit × 1-Digit matching cards (1 set per student pair)

Preview

Say: **Today we will continue to practice using the partial-products method to solve 2-digit by 1-digit multiplication problems.**

Engage Prior/Informal Knowledge — Time: 3 min

Review the break-apart strategy with the basic fact, 4×7. Have students turn to the *Engaged Practice Sheet*.

Say: **On your sheet, draw an array of dots for the multiplication problem, 4×7.**

Now on the grid provided, draw an area model for the same problem, 4×7.

I do not know 4×7 automatically, so I am going to break apart 7 into 2 factors I know. What 2 factors do we break 7 into? *(2 and 5)*

How can we split the array to show the 2 new factors? *(draw a line after the second column)* **Draw it.**

What are the 2 new multiplication sentences for the models? *(4×2 and 4×5)*

What are the 2 partial products to the 2 multiplication problems? *(8 and 20)*

What is the total number of dots in the array? *(28)* **How do you know?** *(added the 2 partial products; counted the dots)*

Does the area model have the same total? *(yes)*

Modeled Practice **Time: 8 min**

1. Estimate a product and use the partial-products method to solve.

   Have students turn to ***Modeled Practice Sheet #1.*** The teacher and students will complete the blanks as the lesson progresses.

Say: **Estimate the product of this multiplication sentence.** *(answers may vary; round 24 to the tens (20), and 8 to the ones (10); 20 × 10 is 200)*

We will review the partial-products method to solve this problem. Today's problem shows a picture of an area model instead of an array. Do you think this will change the steps used in the partial-product method? *(no)*

Look at the area model. *Dimensions* are the length and width of a shape. What are the *dimensions* of a shape? *(length and width)*

What are the *dimensions*, the length and the width, of this rectangle? *(24 by 8)*

We can use the *dimensions* to find the *area*. The *area* is the amount of square units inside of a shape. What is the *area*? *(the square units inside of a shape)*

If we want to find the *area*, the total number of square units inside the rectangle, what would be the equation to solve? *(24 × 8)*

We do not have 24 × 8 memorized, so to solve we will use the partial-products method. When multiplying using the partial-products method, we break apart a factor into tens and ones. Which factor is broken apart into tens and ones? *(24)*

First, break apart 24 into tens and ones. How many tens? *(2 tens)* **What is the value of 2 tens?** *(20)* **How many ones?** *(4 ones)*

How does the area model above represent the breaking apart of 24? *(it split the rectangle by shading the last 4 columns to the right and leaving the first 20 columns to the left white)*

Label the *dimensions*, the length and width, of the new area models. What is the width of the white area model? *(8 square units)* **What is the length of the white area model?** *(20 square units)*

Did the width of the shaded area model change? *(no)*

Did the length of the shaded area model change from the original dimensions of 24? *(yes, it is now 4 squares)*

What are the *dimensions* of the white area model? *(20 by 8)*

What are the *dimensions* of the shaded area model? *(4 by 8)*

What are the 2 multiplication problems for the 2 area models? *(20 × 8 and 4 × 8)*

Watch For

Students may need more concrete examples to see how the total for the area model does not change.
For these students, provide a cutout of the area model. Students can cut the model into two pieces to see how breaking apart the factor does not change the original amount.

Say: **Now that we know the *dimensions* of the 2 new smaller rectangles, we need to multiply to find the partial products.**

What is 20 × 8? Think of 20 as 2 × 10. What is 2 × 8 and then times 10? *(160)*

What is 4 × 8? Use the doubling strategy if needed. *(32)*

What do you do with the 2 partial products? *(add the 2 products)*

What is 160 + 32? *(192)*

What is 24 × 8? *(192)*

Teacher Note

Work with students to add mentally instead of on their paper. Ask what is in the hundreds place in both numbers and add it together. Next, ask what is in the tens place in both numbers and add it together. Finally, ask what is in the ones place in both numbers and add it together. Another suggestion is to ask students to identify the place value that will change and the 2 place values that will stay the same. Only the tens place will change because both numbers have a digit other than 0 in the tens place. The hundreds place and the ones place will not change because only 1 number has a digit other than 0.

Point to the estimated product.

Say: **Is our solution, 192, close to our estimates from earlier?** *(yes)*

2. Estimate a product and use the partial-products method to solve. Have students turn to the ***Modeled Practice Sheet #2***. With the given dimensions, draw in the rectangle on the grid provided. The teacher and students will fill in the blanks as the lesson progresses.

Say: **Let's read the next problem together. Ready? Read: "The ladies' quilting club made a quilt for the auction that sold for $300. The quilt was 27 squares long and 6 squares wide. How many squares were on the quilt altogether?"**

What is the question asking us to find? *(the number of squares on the quilt)* **Underline it.**

What is the important information in the problem? *(27 squares long, 6 squares wide)* **Circle it.**

Do we need to know how much the quilt sold for? *(no)*

In this example there is a grid, but no area model, or rectangle, drawn. Before we solve, first draw the area model using the *dimensions* given in the problem.

What are the *dimensions* of the rectangle we should draw? *(27 by 6)*

The grid's long side goes down, so draw the length of 27 going down the grid and then the shorter width of 6 going across.

First, we will estimate the product of this multiplication sentence. What is your estimation? *(responses may vary; round 27 to the tens (30), and 30 × 6 is 180)*

With the area model drawn, how can we break apart this rectangle to find numbers sentences we can solve mentally? *(draw a line or split it into tens and ones, 20 and 7)*

Why did you choose to split 27 instead of 6? *(separated the tens from ones to provide new multiplication sentences that can be solved mentally)*

Draw a line after row 20. Shade a 7-by-6 rectangle in the bottom section.

Draw in brackets to label the new *dimensions* of each rectangle.

What are the new *dimensions* of this top rectangle? *(20 by 6)*

What are the new *dimensions* of this bottom rectangle? *(7 by 6)*

What is the multiplication problem to find the *area* of the top rectangle? *(20 × 6)*

What is the multiplication problem to find the *area* for the bottom rectangle? *(7 × 6)*

What is 20 × 6? *(120)* **What is 7 × 6?** *(42)* **What strategy can you use to solve 7 × 6 if you don't know it automatically?** *(break 7 into 2 + 5 or break 6 into 1 + 5)*

How do you use both products to solve 27 × 6? *(add the 2 products)*

What is 120 + 42? *(162)* **What is 27 × 6?** *(162)*

162 what? *(squares on the quilt)*

Point to the estimated product.

Say: **Does our solution, 162, come close to our estimate from earlier?** *(yes)*

Did we answer the question? *(yes)* **How do you know**? *(the question was asking for the number of squares on the quilt)*

Practice — Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 30. Students will complete the problems with their partner. Provide corrective feedback as needed.

Say: **Work with your math partner to solve the multiplication problems using the partial-products method. Use the area models to help show how you broke apart a factor.**

Possible questions to check for understanding:

- What is your estimated answer?
- Is your solution close to your estimation?
- How do you plan to break apart the rectangle to find 2 facts you can solve mentally?
- What are the dimensions of the new rectangles after you break them apart?
- What strategy did you use to solve this step (reference a multiplication or addition problem)?

Activity 2: Students will play with a partner using the *2-Digit × 1-Digit matching cards*. With a partner, students will place cards face up in a 3 × 5 array. The first player will try to match 3 cards together. The student is looking for 2 small area models and their corresponding multiplication sentence. If the 3 cards match, the player keeps the cards. The second player then tries to match 3 cards and continues the game. The winner is the player with the most cards.

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work for 5 minutes independently solving multiplication problems, including problems with unknown factors. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 7

Multiplying With Partial Products

<table>
<tr><td rowspan="1">Lesson Objectives</td><td colspan="2">• The student will solve 2-digit by 1-digit multiplication problems.
• The student will follow written steps for the partial-products method in order to solve 2-digit by 1-digit multiplication problems.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">distributive property, partial product</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 70-83)
• Whiteboard with marker
• Partial-Products Method poster
• 2-Digit X 1-Digit and Partial Products matching cards</td><td>• Student Booklet (pp. 36-42)
• Multiplication table (1 per student)</td></tr>
</table>

Preview

Say: **Today we will record our steps in solving 2-digit by 1-digit multiplication problems using the partial-products method.**

Engage Prior/Informal Knowledge

Time: 3 min

Have students turn to the *Engaged Practice Sheet*. Students will evaluate an incorrect answer and determine where the mistake was made.

Say: **The problem on the page is the work of a student who made a mistake. Let's look at what the student did and find the mistake.**

How did the student split the area model? *(split the length, 24 into tens and ones, 20 and 4)* **Is this correct when using the partial-products method?** *(yes)*

What 2 new multiplication problems did the student write to find the partial products? *(20 × 7 and 4 × 7)*

Are these the correct multiplication problems to find the partial products? *(yes)*

Did the student solve the multiplication problems correctly? *(no)* **Why not?** *(20 × 7 is not 1,400, it is 140)*

Okay, so the mistake is a calculation error. Did the student try to complete the rest of the problem correctly? *(yes)* **How do you know?** *(the student added the 2 products together)*

What could the student have done at the very beginning of solving this problem that would have helped them check if the answer was reasonable or not? *(estimate before solving)*

If the student had estimated, what would be the estimated answer? *(20 × 10 = 200)*

Is 1,400 close to 200? *(no)*

Estimation can help us check if our answers are reasonable.

Modeled Practice **Time: 8 min**

1. Students will solve 2-digit times 1-digit multiplication problems without the aid of a visual representation. Problems will be presented horizontally as well as vertically.

   Have students turn to ***Modeled Practice Sheet #1***. The teacher and students will complete the steps together as the lesson progresses.

Say: **We have been working on solving 2-digit by 1-digit numbers using the partial-products method.**

This method is very similar to the fact strategies, breaking apart a factor to find 2 known facts.

Read the first step together. Ready? Read: "Step 1: Estimate an answer." What is step 1? *(estimate an answer)*

We should always estimate an answer first when solving problems with larger numbers. That way we can check that our answer is reasonable, or that it makes sense and is logical.

Let's estimate the product for 39 × 4. *(round 39 to the tens (40); 40 × 4 is 160)* **Write it.**

Read the second step together. Ready? Read: "Step 2: Break apart a factor into tens and ones." What is step 2? *(break apart a factor into tens and ones)*

Which factor in 39 × 4 should we break apart? *(39)*

In previous lessons, we used an array or area model to show the breaking apart of 1 factor into tens and ones. Picture the area model in your mind. How should we break apart 39? *(3 tens (30) and 9 ones (9))* **Write it.**

Why did you choose 30 and 9? *(separated the tens from the ones to provide new multiplication sentences that can be solved mentally)*

Read step 3 together. Ready? Read: "Step 3: Multiply by the other factor." What is step 3? *(multiply by the other factor)*

Think of the picture of the area model we just broke it into 2 smaller area models. What are the 2 new multiplication problems after breaking apart 39 into 30 and 9? *(30 × 4 and 9 × 4)* **Write it.**

What is 30 × 4? *(120)* **Write it.**

What is 9 × 4? *(36)* **Write it.**

Teacher Note
For students who cannot recall their basic facts efficiently, provide a multiplication table.

Say: **These are the partial products. Read the final step. Ready? Read: "Step 4: Add the partial products to find the total. What is step 4?** *(add the partial products)*

What are the products? *(120 and 36)* **What do we do with these products?** *(add them to find the total)*

What is 120 + 36? *(156)* **Write out the addition equation.**

What is 39 × 4? *(156)* **Write it.**

Point to the estimated product.

Say: **Now let's check to see if our answer is reasonable. Is our solution, 156, close to our estimate in Step 1?** *(yes)*

2. Correct a student's mistake when multiplying 2-digit by 1-digit in a vertical format.

Have students turn to ***Modeled Practice Sheet #2.*** The teacher and students will complete the steps together as the lesson progresses.

Say: **We will read the next problem together. The student in the story has found an answer for his problem. We will need to**

check if it is reasonable and check any mistakes the student might have made.

Ready? Read: "Jaime practiced his math facts every day for 8 days. He solved 24 facts each day. How many math facts did he solve in 8 days?"

Jaime worked out the problem, 24 × 8, and got the answer 48. First let's ask ourselves, what is the question asking you to find? *(the number of math facts Jaime solved in 8 days)* **Underline it.**

Next what is the important information? *(24 facts and 8 days)* **Circle it.**

Jaime chose multiplication to solve this problem. Do you think this is the correct operation? *(yes; accept reasonable justifications)*

This time the numbers are written 1 on top of the other, or vertically.

What is the first step in using the partial-products method to solve? *(estimate an answer)* **Did Jaime estimate?** *(no)*

What is a good estimation for 24 × 8? *(20 x 10 = 200 or 25 x 8 = 200; accept other reasonable answers)*

Is Jaime's answer, 48, close to our estimation? *(no)* **Is 48 a reasonable answer for 24 x 8?** *(no)* **Why not?** *(48 is only 24 doubled, the answer should much greater than 48)*

What is the second step in solving this problem using partial products? *(break apart a factor into tens and ones)*

Did Jaime break apart a factor into tens and ones*? (no)*

What factor should we break apart? *(24)* **How can I break apart 24?** *(20 and 4)*

Why do we break the 2-digit number into tens and ones? *(to separate the tens from the ones to provide new multiplication sentences that can be solved mentally)*

What is Step 3? *(multiply by the other factor)*

What are the 2 new multiplication sentences? *(20 × 8 and 4 × 8)* **Write "20 × 8" and "4 × 8" in vertical form.**

What is 20 × 8? *(160)* **Write it.**

What is 4 × 8? *(32)* **Write it.**

What are these 2 numbers called? *(partial products)*

What do we do with these products? *(add the partial products to find the total)* **Write the problem.**

What is 160 +32? *(192)* **Write it.**

What is 24 × 8? *(192)* **Write it.**

Point to the estimated product.

Say: **Is our solution, 192, close to our estimate from earlier?** *(yes)*

192 what? *(math facts)*

Look at Jaime's work. What did he do wrong? *(accept reasonable answers; he did not estimate first; he multiplied 2 × 8, instead of 20 × 8)*

Practice — Time: 8 min

Activity 1: Have students turn to the *Practice Sheet* on page 39. Students will complete the problems with their partner. Provide corrective feedback as needed.

Say: **With your math partner, complete the multiplication problems on your sheet using the partial-products method.**

Fill in the blanks to show the steps you use to find your answer.

Possible questions to check for understanding:

- What are the partial products?

- What is your estimation for this problem?
- What is the best way to break apart the 2-digit number? *(into tens and ones)*
- How does breaking apart 1 of the numbers help you solve the problem? *(separating the tens and ones makes 2 multiplication problems that can be solved mentally)*
- Is your answer reasonable? How do you know? *(it is close to our estimate)*

Activity 2: Have students play *Matching* using the ***2-Digit × 1-Digit and Partial Products matching cards***. Shuffle the ***2-Digit × 1-Digit cards*** and place them face down in a stack on the table. Then deal out the ***Partial Products cards*** to each student until all cards are dealt. Each student should study the cards they were given then lay them face up in front of them.

The first player will turn over the top card from the stack and lay it face up on the table. If they have the matching ***Partial Products card***, the player says "Match" and keeps the card. If someone else in the group has the matching ***Partial Products card***, they must call out "Match" and take the card. If no one calls out "Match" before the next player turns over the next card in the stack, then that card is lost and no one gets it. The player to recognize and call out the most matches wins.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work for 5 minutes independently solving multiplication problems, including problems with unknown factors. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

The Commutative Property of Multiplication With 2-Digit by 1-Digit and Related Division Problems

Lesson Objectives	• The student will use the commutative property of multiplication to solve and write the corresponding division problem for 1-digit by 2-digit multiplication problems. • The student will be able to use content-specific academic vocabulary to example the relationship between 2 multiplication problems and to a related division problem.	
Vocabulary	**number family:** a set of multiplication and division equations made from the same numbers (for 5, 7, and 35, the multiplication/division family consists of 5 x 7 = 35, 7 x 5 = 35, 35 ÷ 7 = 5, 35 ÷ 5 = 7) **corresponding fact:** related facts that are in the same number family	
Reviewed Vocabulary	commutative property of multiplication, dividend, division, factor, partial products method, product	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 84-97) • Whiteboard with marker • Partial-Products Method poster • Go Fish 2-Digit × 1-Digit cards	• Student Booklet (pp. 43-49) • Multiplication table (1 per student) • Partial-Product Method Bookmark (1 per student)

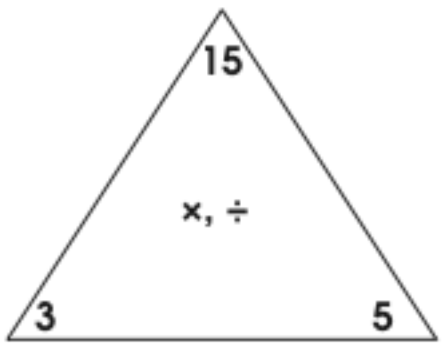
Preview

Say: **Today we will solve sets of multiplication and division problems that are related to one another.**

Engage Prior/Informal Knowledge **Time: 3 min**

Discuss number families and review how multiplication and division are related.

Draw the number family triangle below on the whiteboard.



Ask questions to review vocabulary terms, commutative property, and the corresponding relationships between multiplication and division:

- Write a multiplication sentence from this number family triangle. *(3 x 5 = 15 or 5 x 3 = 15)*
- Remembering the commutative property of multiplication, write another multiplication sentence from this number family. *(3 x 5 = 15 or 5 x 3 = 15)*
- Which number is the product? *(15)*
- Thinking about division, write a division sentence from this number family triangle. *(15 ÷ 3 = 5 or 15 ÷ 5 = 3)*
- Which numbers could be the quotient, or the answer, to a division problem? *(3 or 5)*

- What is the dividend in the division problem? *(15; the number divided by)*
- Would 5 + 3 = 8 be in this number family? *(no)*
- How are the operations of multiplication and division related? *(they are opposites; multiplication is the joining of equal groups and division is the separating of equal groups)*

Modeled Practice — Time: 8 min

1. Review the commutative property of multiplication with larger numbers.

   Have students turn to ***Modeled Practice Sheet #1***. Distribute a ***Partial Products Method Bookmark*** to each student. The teacher and students will complete the steps together at the lesson progresses.

Say: **Use your bookmark to review the steps of the partial products method.**

What is Step 1? *(estimate an answer)*

What is Step 2? *(break apart a factor in tens and ones)*

What is Step 3? *(multiply by the other factor)*

What is Step 4? *(add the partial products to find the total)*

We have a fact to solve. What fact? *(25 × 7)*

What is Step 1 in solving this problem? *(estimate an answer)* **Why?** *(to check that our answer is reasonable)*

How would you estimate this problem? *(we can round; 30 × 7 = 210 or 20 × 10 = 200)* **Write your estimation.**

Teacher Note
Encourage students to be flexible with their rounding. If you round 1 number up, i.e. 25 to 30, you don't also want to round the other number up as well. The estimation would get too high. Allowing 25 to round down to 20 and

then 7 to round up to 10 results in a better estimation.

What is Step 2 of the partial-products method? *(break apart into tens and ones)* **What number do we break apart?** *(25)* **What is 25 broken into?** *(2 tens; 20 and 5 ones; 5)* **Write it.**

What is step 3? (*multiply by the other factor***) What 2 multiplication problems do we solve?** *(20 × 7 and 5 × 7)* **Write it.**

What are the 2 partial products? *(140 and 35)* **Write it.**

What is Step 4? *(add the partial products)* **What is 140 + 35?** *(175)* **Write it.**

What is 25 × 7? *(175)* **Write it.**

Think about *number families,* a set of facts that share the same 3 numbers. What 3 numbers are in this *number family*? *(25, 7, 175)*

What does the commutative property of multiplication allow us to do with the factors? *(change the order of the factors without changing the answer)*

So what is the product of 7 × 25? *(175)*

7 × 25 = 175 and 25 × 7 = 175 are *corresponding facts* because they use the same 3 numbers. These facts make up half of the *number family* for the numbers 7, 25, and 175.

A *number family* is a set of facts that are corresponding because the same 3 numbers are used in each fact.

Write "3 × 4 = 12," 12 ÷ 3 = 4," and "12 ÷ 4 = 3" on the whiteboard.

In the *number family* for 3, 4, and 12, the *corresponding facts* are 3 × 4 = 12, 12 ÷ 3 = 4, and 12 ÷ 4 = 3. What is the fourth fact for this *number family*? *(4 × 3 = 12)*

2. Relate multiplication to the corresponding division facts.

Have students turn to ***Modeled Practice Sheet #2***. The teacher and students will complete the steps together as the lesson progresses.

Say: **Because 7, 25, and 175 are related and we know the *corresponding fact* of 7 × 25 = 175, do we know the corresponding division fact of 175 divided by 7 equals?** *(yes)*

What is 175 ÷ 7? *(25)* **Write it.**

How did you solve that so quickly? *(using knowledge of the number family and the relationship between multiplication and division)*

What other corresponding division fact do we know using these 3 numbers? *(175 ÷ 25 = 7)* **Write it.**

Write "25 ÷ 7 = 175" on the whiteboard.

Say: **Is this number sentence true?** *(no)* **Why not?** *(25 × 7 = 175, not divided by 7; if you have 25 and divided it evenly into 7 groups, there would not be 175 in each group)*

Which number in this *number family* will always be the dividend, or the number being divided? *(175)*

Which number in this *number family* will always be the product, or the answer to a multiplication problem? *(175)*

3. Have students identify corresponding facts and non-corresponding facts from number families.

Have students turn to ***Modeled Practice Sheet #3***. The teacher and students will complete the steps as the lesson progresses.

Say: **We will read the problem together. Ready? Read: "Mr. Perez gave his 36 students 2 facts. It was the students' job to decide if the facts were corresponding or not and then write out the rest of the *corresponding facts* for the *number family*.**

"The first group of students were given the facts 45 × 9 = 405 and 405 ÷ 15 = 27. This group said the facts were corresponding because both facts had 405 as 1 of the numbers. The additional *corresponding facts* they wrote were 9 × 45 = 405 and 405 ÷ 27 = 15."

Are the students correct? *(no)* **Where did they do wrong?** *(the facts are not corresponding because the 3 numbers are not the same; there are 4 numbers involved, not 3)*

What 3 numbers are in the *number family*? *(45, 9, 405)* **What would be the corresponding division facts for the 2 multiplication facts?** *(405 ÷ 9 = 45 and 405 ÷ 45 = 9)* **Write it.**

What would be the corresponding multiplication facts for the 2 division facts? *(15 × 27 = 405 and 27 × 15 = 405)* **Write it.**

What are the 3 numbers in these *corresponding facts number family*? *(15, 27, and 405)*

Practice — Time: 8 min

Activity 1: Students will work with a math partner to complete the *Practice Sheet*.

Have students use their *Partial Products Method Bookmark* during the practice section.

Say: **Work with your math partner to complete the problems on your sheet. Refer to your bookmark for the steps to solving using partial products.**

Remember the *corresponding facts* within a *number family*.

Ask questions such as:

- Why do you estimate before solving? *(to check if my answer is reasonable)*

- Do you need to use the partial-product method to solve problem #2? *(no)* Why not? *(the commutative property of multiplication states that switching the factors does not change the answer)*
- Which number will be the dividend in the division sentences? *(64)*
- Explain how multiplication and division are related. *(multiplication is combining equal-sized groups, while division is separating equal-sized groups)*

Activity 2: Students will play *Go Fish* using the ***Go Fish 2-Digit × 1-Digit cards*** provided. First, shuffle the cards and deal each student 5 cards. Students take turns asking each other for a number family, e.g.,"Billy, do you have the number family 4, 52, and 208?" If the student has a multiplication or division sentence that is a part of that number family, the player will surrender the card(s). If they do not, they will reply, "Go fish." The player asking will draw a card from the draw pile. Normal *Go Fish* rules should apply. Once a student collects all 4 cards in that fact family, the student may lay it down as a "book." Students continue playing until all books are made or until time has run out.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheet*** and complete as many items as possible.

Say: **You will work for 5 minutes on your own. Complete as many problems as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 9

Division as Equal Sharing

<table>
<tr><td>Lesson Objectives</td><td colspan="2">• The student will use base-10 materials to solve 2-digit by 1-digit division problems.
• The student will use terminology associated with division during discussion.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">equal share: the share found by breaking quantities apart so that everyone gets the same amount</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">division, equation, factor, number family, product</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 98-107)
• Base-10 materials: 18 ones
• 20 to 25 items (counters, pencils, or markers) for students to share</td><td>• Student Booklet (pp. 50-54)
• Base-10 materials: 20 ones (1 set per student pair)
• Whiteboard with marker (1 per student)</td></tr>
</table>

Preview

Say: **Today we will practice division problems with larger numbers that we do not know automatically. We will use *equal shares* to determine the same amount each group receives.**

Engage Prior/Informal Knowledge — Time: 3 min

Present questions to activate students' background knowledge or prerequisite skills related to equal sharing.

Have students explain in their own words:

- Think about sharing. What does it mean to share equally? *(to break a quantity apart so that everyone gets the same amount)*
- How would you share 21 candy bars between 2 friends? 4 friends? 8 friends? *(answers will vary)*
- What happens to the amount that we cannot share equally? *(it is left over; it cannot be shared equally)*

Modeled Practice — Time: 8 min

1. Using base-10 materials, model the concept of equal shares as division for 3 cups.

   On the ***Modeled Practice Sheet #1,*** place 18 ones in the top box.

   Have students help you divide 18 ones among 3 cups, 6 cups, and then 9 cups. Have students turn to ***Modeled Practice Sheet #1.*** Fill in the "Total," "Cups," "Equal Share," and "Left Over" lines as the lesson progresses. First, point to the row of 3 cups on the sheet.

Say: **Division can be thought of as *equal sharing*. An *equal share* is when we break a quantity or a group apart so that everyone gets the same amount.**

What is an *equal share*? *(when we break a quantity apart so that everyone gets the same amount)*

Let's read the first problem together. Ready? Read: "Lauren was asked to fill 3 cups with ice cubes at the lemonade stand. She counted 18 ice cubes in her bucket. If Lauren places the same number of ice cubes in each cup, how many ice cubes will be in each cup?"

What is the question asking you to find? *(number of ice cubes per cup)*

How many ice cubes to start? *(18)* **Place 18 ones in the box. Let's pretend these are ice cubes. We need to share these ice cubes with the 3 cups.**

What is the total number of ice cubes we have? *(18)* **Write it on the "Total" line.**

How many cups are sharing the ice cubes? *(3)* **Write it on the "Cups" line.**

We need to *equally share* 18 ice cubes among 3 cups. What does that mean? *(we have to give each cup the same amount of ice cubes until there are no more ice cubes left to share)*

To find out how many ice cubes each cup receives, we have to distribute the ice cubes so that each cup gets an equal amount. What is this example of? *(an equal share, or dividing)*

Begin sharing the ice cubes among the 3 cups.

Say: **If we give 1 ice cube to 1 cup, how many ice cubes do we give to the other cups?** *(1)* **Why?** *(equal share means everyone gets the same amount)*

Have a student volunteer help to continue sharing the ones between the 3 cups, moving the ones to the cups as they are shared.

Say: **How do we know when we are finished *equally sharing*?** *(when no ice cubes are left to share or when I can't give the same amount of ice cubes to each cup)*

Are all the ice cubes shared equally among the cups? *(yes)* **Do we have any left over, or ones that cannot be shared equally among the cups?** *(no)*

How many ice cubes does each cup get? *(6)* **Write it on the "Equal Share" line.**

Did all the cups get the same or equal amount? *(yes)* **Because all the cups have the same amount, the ice cubes have been *equally shared*.**

If we had 1 ice cube left over, we couldn't give it to any of the cups, because then it would not be equal. The 1 ice cube would be left over. Do we have any left over ice cubes? *(no)* **What number do we write to show there are no ice cubes left over?** *(0)* **Write it on the "Left Over" line.**

Read the division equation. *(18 divided by 3 equals 6)*

2. Using base-10 materials, model the concept of equal shares as division for 6 cups.

   Have students turn to ***Modeled Practice Sheet #2***. Replace the 18 ones in the box at the top of the page. Point to the group of 6 cups.

Say: **Can you predict what would happen to the *equal share* of ice cubes if more cups were put out to share the same 18 ice cubes?** *(answers will vary; each cup's share would get smaller).*

How many ice cubes total? *(18)* **Write it on the "Total" line.**

How many cups are sharing the ice cubes now? *(6)* **Write it on the "Cups" line.**

How can we find out how many ice cubes each cup receives? *(share the ice cubes equally between the 6 cups)*

Can we give 2 ice cubes to the first 4 cups and the rest to the fifth and sixth cups? *(no)* **Why?** *(equal sharing means everyone gets the same amount)*

Let's continue equally sharing the ice cubes among the 6 cups. How do we know when we are finished sharing? *(when no ice cubes are left to share or when I can't give the same amount of ice cubes to each cup)*

Do we have any ice cubes left over when we shared 18 among 6 cups? *(no)*

How many ice cubes does each cup get? *(3)* **Write it on the "Equal Share" line**

How many are left over? *(0)* **Write it on the "Left Over" line.**

What happened to each *equal share*? *(it got smaller)* **Why?** *(the total stayed the same, but the number of cups sharing got bigger)*

Because all cups have the same amount of ice cubes, the ice cubes have been shared equally.

Read the division equation. *(18 divided by 6 equals 3)*

How is the first problem related to the second problem? *(they are in the same number family; the equations are corresponding)*

3. Using base-10 materials, model the concept of equal shares as division for 8 cups.

   Replace the 18 ones to the box at the top of the page. Point to the group of 8 cups.

Say: **How many cups are sharing the ice cubes now?** *(8)* **Where do we write it?** *(on the "Cups" line)*

How can we find out how many ice cubes each cup receives? *(share the ice cubes equally among the cups)*

Did the total change? *(no)* **How many total ice cubes?** *(18)* **Where do we write it?** *(on the "Total" line)*

Have a student volunteer share the ones between the 8 cups, moving the ones to the cups as they are shared. Stop when 2 ones are left.

Say: **How do we know when we finished sharing?** *(when no ice cubes are left to share or when I can't give the same amount of ice cubes to each cup)*

We have 2 ice cubes left to share. We can't break them apart. Can we share equally? *(No)* **Why?** *(you only have 2 ice cubes and there are 8 cups)*

These 2 ice cubes are "left over."

How many ice cubes does each cup get? *(2)* **Where do we write it?** *(on the "Equal Share" line)*

How many are left over? *(2)* **Where do we write it?** *(on the "Left Over" line)*

Because all cups have the same amount of ice cubes, the ice cubes have been shared equally. Because we have ice cubes that could not be shared, we place those ice cubes in the "left overs."

Read the division equation. *(18 divided by 8 equals 2 with 2 left over)*

What happened to the *equal share* as we added more cups? *(it got smaller)* **Why?** *(the total stayed the same, but the number of cups sharing got bigger)*

Can you predict what might happen to the amount each cup gets if I keep adding cups? *(it would get smaller)*

Practice **Time: 8 min**

Activity 1: The students will share items given by the teacher and write a division equation for that they did.

Place 20 to 25 of the same items (counters, pencils, or markers) on the table for the students to share. Make sure the solution will include left over items. Distribute a whiteboard with marker to each student.

Teacher Note
You can also continue the use of the base-10 ones instead of another form of counters for this activity.

Say: **As a group, we need to share these items. Let's work together to decide how much each person will receive. Then, we will write on our whiteboards the division equation for what we did.**

Ask the following questions:

- How many "items" did you get? Is this the same amount as [student name]?
- Are the items shared equally? How could we share the items equally?
- Are there any items left over?

Activity 2: Students will complete the ***Practice Sheet*** on page 52 with their partner. Have students use ones to solve. Students will distribute the ones to equally share among the varying amounts of cups.

Say: **With a math partner, use the ones to solve the division problem on the sheet.**

Select a few students to verbalize their reasoning, asking the following questions:

- How did you share the ones qually? *(allow a variety of answers)*

- How do you know when you have shared the ones equally? *(when they are all gone and each cup has the same amount)*
- If I add more cups, what will happen to the amount of ones in each cup gets? *(the amount is less per cup)*

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible. Have students use ones to represent the bananas being shared.

Say: **You will work independently for 5 minutes. You will use ones to help you find the solution. Complete as many as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multipliation & Division of Whole Numbers
Lesson 10

Division as Equal Sharing Using Tens and Ones

Lesson Objectives	• The student will solve 2-digit by 1-digit division problems using tens and ones. • The student will correctly use terminology for division during discussions.	
Vocabulary	**efficient:** a faster way to complete a task **remainder:** the amount left over when things are divided into equal shares **quotient:** the factor solved for in a division problem (for example, in $15 \div 3 = 5$, 5 is the quotient, or equal share) **dividend:** in division, the number that is being divided (for example, in $15 \div 3 = 5$, 15 is the dividend) **divisor:** in division, the number that divides another number, the dividend (for example, in $15 \div 3 = 5$, 3 is the divisor)	
Reviewed Vocabulary	division, equal share	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 108-125) • Base-10 materials: 5 tens and 50 ones • Whiteboard with marker	• Student Booklet (pp. 55-63) • Base-10 materials: 8 tens and 15 ones (1 set per student pair)

Preview

Say: **We will continue to study division through equal sharing. Today we will break apart our total into tens and ones to share equally and more *efficiently*.**

Engage Prior/Informal Knowledge **Time: 3 min**

Review decomposing numbers into tens and ones.

Have students turn to the ***Engaged Practice Sheet***. Students will decompose the numbers into tens and ones.

Say: **On the sheet, decompose the numbers into tens and ones.**

Break apart 98 into tens and ones. How many tens? *(9)* **How many ones?** *(8)*

Ask the following questions:

- How could we write 74 as tens and ones? *(7 tens and 4 ones)* 56? *(5 tens and 6 ones)*
- Is 7 tens and 4 ones the same as 74? *(yes)* How do you know? *(accept reasonable answers; 7 is in the tens place, which represents 7 tens, and so on)*

Modeled Practice **Time: 8 min**

1. Using base-10 materials, model the concept of equal shares as division by breaking numbers into tens and ones.

   Count out 43 ones and display them for students. Have students turn to ***Modeled Practice Sheet #1***. Point to the ones and tens on the sheet. The teacher and students will complete the steps together as the lesson progresses.

Say: **I have counted out 43 ones. Pretend these ones are marbles. We are going to equally share these marbles. How many total marbles?** *(43)* **Write "43" on the "Total" line.**

Look at the people on your sheet. How many people are sharing the marbles? *(3)* **Write "3" on the "# of People" line.**

We need to equally share 43 marbles between 3 people. What does it mean to equally share? *(you have to give the same amount of marbles to each person until there are none left that can be shared)*

We could use all of these ones to represent the 43 marbles. I am wondering if there is another way we could represent 43.

How many tens are in 43? *(4)* **Write "4" on the "Total Tens" line. How many ones are in 43?** *(3)* **Write "3" on the "Total Ones" line.**

Replace the 43 ones with 4 tens and 3 ones.

Say: **Using tens and ones to represent 43 will help us share the marbles more *efficiently*. *Efficient* means a faster way to solve. What does *efficient* mean?** *(to do something faster)*

To find the equal share, we have to give out the marbles so that each person gets the same amount. We can do this using the tens and ones.

Point to the 4 tens.

Say: **Could we equally share the marbles in groups of 10, meaning can we give out 10 marbles at 1 time to each person?** *(yes)* **Why?** *(there are enough groups of 10 to give each person 1)* **If we give 1 group of 10 marbles to 1 person, how many groups of 10 do we give to the others?** *(1)* **Why?** *(we have to equally share all marbles with all the people)*

Have student volunteers continue sharing the tens between the people by moving the tens to the bags as they are shared until there is 1 group of 10 left.

Say: **How many marbles are left?** *(13)* **Are we finished equally sharing?** *(no)* **How do you know?** *(there are still marbles that I can share equally)*

Give 1 person the last ten.

Say: **Can we give 1 person this group of 10?** *(no)* **Why?** *(it wouldn't be equal)* **How many marbles does this 1 person have?** *(20)* **How many marbles does everyone else have?** *(10)* **Is that equal?** *(no)*

Remove the 1 ten. Have a student volunteer replace 1 ten for 10 ones. Move all ones into 1 group. Demonstrate 1-for-1 sharing.

Say: **We can share the 1 group of 10 by decomposing, or breaking, the group of 10 into ones. How many ones are in 1 group of 10?** *(10)*

We continue sharing the marbles. How do I know when I am finished sharing? *(when no marbles are left to share or when I can't give the same amount of marbles to each person)*

Have a student volunteer distribute the ones to each person. Point to the remaining ones.

Say: **We have 1 left to share. If we don't break it apart, can we share it equally?** *(no)* **Why?** *(you only have 1 marble and there are 3 people)*

Let's count the total number of marbles each person gets. How many groups of ten? *(1)* **Write "1" on the "Tens Share" line.**

How many ones did each person get? *(4)* **Write "4" on the "Ones Share" line.**

How many total marbles does each person get? *(14)* **Write "14" on the "Equal Share" line.**

Why is 14 an equal share? *(because everyone got the same amount)*

We have 1 marble left over. Where do we put this marble that we could not share? *(in the left overs)*

The mathematical word for "left over" is *remainder*. The *remainder* is the amount left over when things are divided into equal shares.

What is a *remainder*? *(the amount left over when things are divided into equal shares)*

What is the *remainder* for this problem? *(1)* **Write "1" on the "Remainder" line.**

2. Using the base-10 picture, model the concept of equal shares as division by breaking numbers into tens and ones.

Say: **Let's use tens and ones to solve another equal sharing problem.The number in the problem will be represented with a picture instead of the base-10 materials.**

Have students turn to *Modeled Practice Sheet #2*. The teacher and students will complete the steps together as the lesson progresses.

Say: **How many marbles total are being shared?** *(73)*

The mathematical word we use for the total amount being divided or shared is the *dividend*. What is the *dividend*? *(the total amount being divided or shared in a division problem)*

How many tens are in the *dividend*? *(7)*

How many ones in the *dividend*? *(3)*

Write "7" on the "Total Tens" line, "3" on the "Total Ones" line, and "73" on the "Dividend (Total)" line.

How many bags are there? *(6)*

We can think of these 6 bags as sharing the total amount of marbles. When we talk about the number that divides another number, or the number of people or bags in a division problem, we call that the *divisor*. What is the *divisor*? *(the number that divides another number)*

Write "6" on the "Divisor" line.

Instead of using the materials of tens and ones, we can draw the amounts to be shared. When drawing a group of 10, draw a straight line. How do we show a group of 10? *(draw a straight line)*

To draw ones, draw a quick circle. How do we show the ones? *(draw a quick circle)*

Draw the 7 tens and 3 ones.

Let's now share the tens among the bags, crossing out the groups of 10 as they are shared.

Continue until there is 1 ten left to share.

Say: **Are we finished equally sharing?** *(no)* **Why?** *(there are still marbles that I can share equally)* **There is only 1 ten left. Can we share this group of 10?** *(yes)* **How can we continue equally sharing?** *(break the 10 into ones)*

We can share the 1 group of 10 by decomposing or breaking apart the group of 10 into ones. How many ones are in 1 group of 10? *(10)*

Trade the group of 10 for 10 ones. Cross out the last ten and draw 10 circles. Are there enough ones to continue equal sharing? *(yes)*

Share the ones between the bags, crossing out the ones as they are shared.

We continue sharing the marbles, or the ones. How do we know when we are finished sharing? *(when no marbles are left to share or when I can't give the same amount of marbles to each person)*

We have 1 marble, or 1 ones, left to share. If we don't break it into smaller pieces, can we share it equally? *(no)* **Why?** *(you only have 1 marble and there are 6 bags)*

How many groups of 10 marbles pieces does each bag get? *(1)* **Write "1" on the "Tens Share" line.**

How many ones does each bag get? *(2)* **Write "2" on the "Ones Share" line.**

How many marbles does each bag get in all? *(12)* **Write "12" on the "Quotient (Equal Share)" line.**

When we talk about the equal share, or the answer, we call that the *quotient*. What is the *quotient*? *(the answer to a division problem)*

Because all bags have the same amount of marbles, the marbles have been shared equally. Because we have 1 marble that could not be shared, we place that marbles in the left overs as the *remainder*.

Where do we put this amount that we could not share? *(in the left overs)*

What is the mathematical word for left over? *(remainder)*

What is the *remainder*? *(1)* **Write "1" on the "Remainder" line.**

Practice **Time: 9 min**

Activity 1: Students will complete the *Practice Sheet* on pages 58 and 59. Work as a group to solve. Have the students draw base-10 pictures to represent the problem.

Say: **Let's read the problem together. Ready? Read: "Janice filled 6 baskets with equal amounts of biscuits. She had 74 biscuits to share among the baskets. How many biscuits did Janice place in each basket?"**

What is the question asking you to find? *(how many biscuits in each basket)* **Underline it.**

What is the important information? *(6 baskets and 74 biscuits)* **Circle it.**

What mathematical operation will we use to solve? *(division)*
How do you know? *(the story says to fill each basket with equal amounts; the story gave us the whole and 1 part and asked us to find the other part)*

Draw 74 in base-10 pictures and then use the picture to help you solve.

As students work through the problem, ask questions such as:

- Did you have to trade any tens for ones? *(yes)*
- What is the approach (steps) to share the biscuits equally? *(draw 74 in base-10 pictures, give each person 1 tens at a time, trade any left over tens for ones, then give each person 1 one at a time until all has been shared that can be shared; if any are left over that cannot be shared equally, put those in the leftovers)*
- How do you know when you have shared the biscuits equally? *(no biscuits are left to share or when I can't give the same amount of biscuits to each person)*
- What is the dividend? *(74)* The divisor? *(6)* What is the remainder? *(2)*
- How many biscuits did Janice place in each basket? *(12)* How many did she have left over? *(2)*

Activity 2: Students will complete the ***Practice Sheets*** on page 60. Working with a partner, have students use the base-10 picture to solve. Then, students will draw base-10 pictures—straight lines for tens and quick circles for ones—to share the different amounts of marbles.

Say: **With a math partner, use the base-10 picture that is shown or draw your own base-10 picture to solve the division problems.**

Select a few students to verbalize their reasoning, asking the following questions:

- What is the approach (steps) to sharing the marbles equally? *(draw 68 in base-10 pictures; give each person 1 tens at a time; trade any left over tens for ones, then give each person 1 ones at a time until* *all has been shared that can be shared; if any are left over that cannot be shared equally, put those in the leftovers)*

- How do you know when you have shared the marbles equally? *(when no marbles are left to share or when I can't give the same amount of marbles to each person)*
- What do we do with the marbles that we could not share? *(keep as the remainder)*
- What is the dividend? The divisor? The remainder?

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible. Have students use base-10 materials to complete the first problem. Then, remove the materials so students can complete the rest of the sheet without the use of manipulatives.

Say: **You will work independently for 5 minutes. You may use base-10 materials on the first problem only. When you finish with the first problem, please pass in your base-10 materials. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 11

Estimating the Answer to Division Problems

Lesson Objectives	• The student will estimate solutions to division problems. • The student will verbalize which estimated number is closest to the dividend and why.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	dividend, division, divisor, equal share, estimation, multiple, quotient, remainder, rounding	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 126-141) • Whiteboard with marker	• Student Booklet (pp. 64-71) • Whiteboard with marker (1 per student) • Multiplication Table (1 per student) • Base-10 materials: 3 tens and 10 ones (1 set per student)

Preview

Say: **Today we will estimate solutions to division problems using estimation and corresponding multiplication facts.**

Engage Prior/Informal Knowledge — Time: 3 min

Review multiples and solving division problems by thinking of the missing factor in a multiplication problem.

Have students turn to the *Engaged Practice Sheet*. Distribute the *Multiplication Table* to each student.

Say: **Look at the multiplication table. Which numbers are the factors?** *(the numbers in the top row and far left column)*

Which numbers are the multiples? *(all the numbers in the table, except the top row and far left column)*

Complete the sheet using your multiplication table.

Have students complete the sheet, then review the answers.

Modeled Practice — Time: 8 min

1. Use estimation and multiplication to solve unknown division problems.

   Have students turn to *Modeled Practice Sheet #1*. The teacher and students will complete the steps together as the lesson progresses.

Say: **To solve division problems, we can think of them as multiplication with a missing factor, or an unknown. For example, $30 \div 6 = n$ can be thought of as $6 \times n = 30$. What is 30 divided by 6 or what number times 6 equals 30?** *(5)*

What is 45 divided by 5, or what number times 5 equals 45? *(9)*

We can use our knowledge of multiplication to help solve division.

Read the next problem. *(34 divided by 5)*

Think about multiplication and multiples of 5. Is 34 a multiple of 5, or can 5 times another number equal 34? *(no, it is not a multiple)*

34 is not a multiple of 5, so we cannot equally share 34 in 5 groups. When we cannot equally share an amount, there will be a remainder.

Rather than working out this problem with long division, we can use estimation to get an answer that is close, but not exact.

List the multiples of 5 up to 50. *(5, 10, … 50)*

34 is between which 2 multiples of 5? *(30, 35)* **Write down the 2 estimation division problems.** *(30 ÷ 5 and 35 ÷ 5)*

Teacher Note
If students have trouble with basic facts, allow students to use the ***Multiplication Table.***

Say: **Write them on the lines for the estimated division problems.**

What is 30 ÷ 5, or what number times 5 is 30? *(6)* **Write it.**

What is 35 ÷ 5, or what number times 5 is 35? *(7)* **Write it.**

Because 34 falls between 30 and 35, what does this tell us about the answer to our division problem, 34 divided by 5? *(the answer is more than 6, but less than 7, or the answer is between 6 and 7)*

How can an answer be in between 2 whole numbers? *(it has a remainder)*

Which multiple of 5 is 34 closer to, 30 or 35? *(35)* **Will the answer to 34, divided by 5, be closer to 6 or 7?** *(closer to 7)* **Write “7” in the space above “is about” to complete the number sentence.**

The wavy equal sign shows the answer is an estimation and not the exact answer.

2. Use estimation and rounding to estimate 3-digit by 1-digit quotients.

   Have students turn to ***Modeled Practice Sheet #2***. The teacher and students will complete the steps together as the lesson progresses. Point to the problem, 255 ÷ 4, on the sheet.

Say: **Read the next problem.** *(255 divided by 4 equals f or f times 4 equals 255)*

"*f*" is our unknown.

Instead of working out this division problem with long division, we can use estimation to find about how much 255 divided by 4 is.

We need to find a number that is close to 255 that can be divided by 4 and would not have a remainder.

255 is a 3-digit number. To estimate this problem using mental math, we can just look at the first 2-digits. What are the first 2-digits? *(25)*

Think about your 4s facts and multiples of 4. Let's ask ourselves: 4 times what number equals a product close to 25?

What are some multiples of 4 that are close to 25? *(20, 24, 28)*

Which multiple of 4 is the closest to 25? *(24)* **Write the division problem we can solve that is closest to the original problem.**

What number times 4 equals 24? *(6)* **Write it.**

However, we are not solving 25 ÷ 4, we are solving 255 ÷ 4. What number times 4 equals 240? *(60)* **How do you know?** *(240 is 10 times greater than 24, and 60 is 10 times greater than 6)*

Is 240 close to 255 and a multiple of 4? *(yes)* **Write the estimated division problem, "240 ÷ 4 = 60."**

What is the next multiple of 4 that is close to 25, 20, or 28? *(28)* **Why 28 and not 20?** *(24 is the next multiple that is less than 25 and 28 is next multiple greater; 20 is further away than 28)* **Write it.**

What number times 4 equals 28? *(7)* **Write it.**

Think about the original division problem, 255 ÷ 4. We are really looking for what number times 4 equals 280, not just 28.

What number time 4 equals 280? *(70)* **How do you know?** *(280 is 10 times greater than 28 and 70 is 10 times greater that 7)*

Write "280 ÷ 4 = 70" on your sheet.

What 2 estimated numbers does the answer fall between? *(60 and 70)* **Is the actual answer closer to 60 or 70?** *(60)* **How do you know?** *(255 is closer to 240 than 280)*

So the actual answer to 255 divided by 4 would be close to 60, because the estimated answer is 60. Write "60" in the space below "is about" to complete the number sentence.

Practice — Time: 8 min

Activity 1: Students will complete the *Practice Sheets* on pages 67 and 68 with their partner.

Say: **We will complete this first problem together.**

Let's read the problem. Ready? Read: "Bridget collected 295 signatures for the petition. If she collected about the same number of signatures each day for 7 days, about how many signatures did we get each day?"

What is this question asking us to find? *(the number of signatures collected in one day)*

Is this question asking for an exact answer or an estimated answer? *(estimated)* **How do you know?** *(it says "about")*

What is the mathematical problem we need to write and solve for? *(295 ÷ 7 = n or n x 7 = 295)*

With your math partner, estimate the answer to the division problem. Remember to find the 2 closest multiples that answer what would fall between.

After students have completed the problem, have them share their answers and their reasoning. Ask question such as:

- What 2 multiples of 7 does 29 fall between? *(28 and 35)*
- What 2 multiples of 7 does 295 fall between? *(280 and 350)*
- What are the 2 estimated answers the actual answer would fall between? *(40 and 50)*
- Is the actual answer closer to 40 or 50? *(closer to 40)* How do you know? *(295 is closer to 280 than 350)*
- About how many signatures did Bridget get each day for the petition? *(about 40 signatures)*

Say: **Work with your math partner to complete the page.**

Select a few students to verbalize their reasoning, asking the following questions:

- 56 ÷ 6:
  - o What multiplies of 6 are close to 56? *(54 or 60)*
  - o Which estimated number is closest to 56? *(54)*
  - o How did you decide which estimated number to use? *(answers may vary; 56 is only 2 away from 54)*
- 370 ÷ 9:
  - o What multiplies of 9 are close to 37? Are close to 370? *(36 and 360 or 45 and 450)*

- Which estimated number is closest to 370? *(360)*
- How did you decide which estimated number to use? *(allow a variety of answers; I know that 9 x 4 is 36 or 9 x 5 is 45)*

Activity 2: Students will earn points by quickly finding multiples of numbers using a ***Multiplication Table***.

Display the following problems on a whiteboard:

$750 \div 9$ $426 \div 5$ $387 \div 4$ $278 \div 7$

Ask students to first look at the hundreds and tens place or a 2-digit number to find the estimated number to the 1-digit divisor. Then, expand the 2-digit number to find the estimated number of the 3-digit dividend using multiples and knowledge of the multiplication facts. Award students 1 point for locating an estimated number or award them 2 points if their estimated number is closer to the actual number in the problem. For example, a multiple of 9 for 75 (750) would be 7 (70), 63 (630), but a closer estimate would be 8 (80), 72 (720).

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Teacher Note
If students have trouble with basic facts, allow them to use a multiplication table.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 12

Division as Equal Sharing Using Tens and Ones With Estimation

Lesson Objectives	• The student will estimate solutions to division problems. • The student will solve 2-digit by 1-digit division sentences using tens and ones. • The student will recognize words and phrases in discussion.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	dividend, division, divisor, efficient, quotient	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 142-155) • Whiteboard with marker • Base-10 materials: 5 tens and 51 ones • Estimation cards	• Student Booklet (pp. 72-78) • Base-10 materials: 7 tens and 10 ones (1 set per student pair)

Preview

Say: **Today we will estimate answers to division problems and then solve through equal shares of tens and ones.**

Engage Prior/Informal Knowledge — Time: 3 min

Review estimation.

Write the following division problems 1 at a time on a whiteboard and ask the questions below:

31 ÷ 4 17 ÷ 6 33 ÷ 7

- What 2 multiples of the divisor [4, 6, 7] are close to the dividend [31, 17, 33]? *(28 and 32; 12 and 18; 28 and 35)*
- Which of the 2 estimated multiples is closer to the dividend? *(32; 18; 35)*
- What number is the actual answer closest to? *(8; 3; 5)*

Modeled Practice — Time: 8 min

1. Use base-10 materials to estimate the solution to a division problem.

   Count out 51 ones and display them for students. Point to the ones and to the ***Modeled Practice Sheet.*** The teacher and students will complete the steps together as the lesson progresses.

Say: **Pretend these ones are marbles. There are 51 total marbles.**

How many bags do we have to divide the marbles into? *(4)*

We need to equally share 51 marbles among 4 bags. What does that mean? *(you have to put the same amount of marbles in each bag until there are none left that can be shared)*

Before we share the marbles, let's estimate to see about how many marbles will be in each bag.

First, is 51 a multiple of 4? *(no)* **51 is not a multiple of 4, so we will need to find 2 multiples that 51 is between.**

We can estimate how many marbles each bag will get by finding a multiple of 4 that is close to 51. What numbers are close to 51 and a multiple of 4? *(48 and 52 are multiples of 4)*

Write "48" and "52" on the "Estimation" line.

Teacher Note
Students may have trouble determining multiples of a number past 12. If so, have students determine the largest multiple they know (such as 4 x 10 or 4 x 12) and list the multiples following that until they reach a number that is close to the dividend.

Say: **Which multiple, 48 or 52, would give a better estimate of 51 divided by 4?** *(52)* **Why?** *(because it is closer to 51 than 48 is)*

What is 52 ÷ 4, or what number times 4 equals 52? *(13)* **Write it.**

Our estimated answer for 51 divided by 4 is 13 because the closest multiple of 4 is 52.

Let's see if our actual answer is close to 13 using equal sharing. Write "13" on the line above "is about" in the last number sentence.

2. Calculate the exact answer to the division problem using equal sharing of tens and ones. Use the ***Modeled Practice Sheet*** and base-10 materials tens and ones.

   Refer to the ones representing the 51 marbles.

Say: **We could use all of these ones to represent the 51 marbles. What is another, or more efficient, way to represent 51 marbles using base-10 materials?** *(with 5 tens and 1 one)*

Replace the 51 ones with 5 tens and 1 ones.

Say: **Write "5" on the "Total Tens" line and "1" on the "Total Ones" line. What is the dividend in this problem?** *(51)* **Write it on the "Dividend" line.**

What number is the divisor, the number of bags we are dividing by? *(4)* **Write it on the "Divisor" line.**

Using tens and ones to represent 51 will help us share the marbles more efficiently. Remember, efficient means a faster way to solve.

How do I find an equal share? *(give out the marbles so each bag gets the same amount)*

Point to the 5 tens.

Say: **If we give 1 group of 10 marbles to 1 bag, how many groups of 10 do we give to the others?** *(1)* **Why?** *(so every bag gets the same amount)*

Have a student volunteer help to share the tens between the bags, moving the tens to the bags as they are shared.

Say: **How many groups of 10 marbles does each bag get?** *(1)* **Write "1" on the "Tens Share" line.**

Are we finished equal sharing? *(no)* **Why?** *(there are still marbles that can be shared equally)*

What can I do with this last group of 10? *(replace it with 10 ones)*

Have a student volunteer replace 1 tens for 10 ones. Move all ones into 1 group.

Say: **How many ones do we have now?** *(11)*

What should we do next? *(share the marbles equally)*

Have a student volunteer move 2 ones to each bag. Point to the remaining 3 ones.

Say: **We have 3 left to share. Can I share them equally?** *(no)* **Why?** *(you only have 3 marbles and there are 4 bags)*

We have 3 marbles left over. Where do we put what could not be shared? *(in the left overs or remainder)*

How many single marbles, or ones, does each bag get? *(2)* **Write "2" on the "Ones Share" line.**

How many groups of 10 marbles does each bag get? *(1)*

How many total marbles does each bag get? *(12)* **Write "12" on the "Quotient" line.**

Because each bag has the same amount of marbles, the marbles have been shared equally. Because we have 3 marbles that could not be shared, we place those marbles in the left overs or remainders. Write "3" on the "Remainder" line.

3. Compare the estimated answer from Step 1 to the answer found by equal shares in Step 2. Continue on the ***Modeled Practice Sheet.***

Say: **When we estimated the answer to 51 divided by 4, what number did we get?** *(13)*

Is this the same as the answer we found using equal sharing? *(no, we got 12 with 3 left over)*

When dividing to find the actual answer, we got 12 marbles with 3 left over.

Was the estimated answer over or under the actual answer? *(over)* **How do you know?** *(13 is more than 12)*

Teacher Note
Remind students that with estimation it is okay if the answer is sometimes over the actual answer. Emphasize the fact that the answer is reasonable.

Say: **How many more marbles would we need so that every bag gets 13 marbles?** *(1 more)*

If we had 1 more marble, we would have been able to equally share all the marbles with 4 bags, giving each bag 13 marbles. So, 12 with 3 left over is *almost* 13 marbles per bag. This means that our estimate was very close to the actual answer.

Practice

Time: 8 min

Activity 1: Have students turn to the ***Practice Sheets*** on pages 73 and 74. Students will complete the problems with a partner. Provide corrective feedback as needed. Have tens and ones available for students to use.

Say: **Work with the base-10 materials to solve the division problems.**

Teacher Note
Allow students to draw base-10 pictures instead of using the base-10 materials provided if students are comfortable with drawing.

Possible questions to check for understanding:

- What is your estimated answer?
- What 2 estimated multiples did you use?
- Is your solution close to your estimation?
- What steps did you use to share the [object] equally?
- What is the dividend? The divisor?

Activity 2: The teacher passes out 1 ***Estimation card*** to each student. The students take turns reading their cards to the group. The group will work together to form the estimated problem, then predict if the estimation will be over or under the actual answer. Each student then votes with a thumbs up for over the actual answer and a thumbs down for under the actual answer. The actual answer, provided on the back of the card, is read by the student. The student who makes the greatest number of correct predictions wins.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible. Have tens and ones available.

Say: **You will work independently for 5 minutes. Complete as much as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

ESTAR INTERVENTION

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 13

Using Area Models to Solve 2-Digit by 2-Digit Multiplication Problems

<table>
<tr><td>Lesson Objectives</td><td colspan="2">• The student will use an area model to reinforce the partial-products method with 2-digit by 2-digit numbers.
• The student will state the similarities between the partial products method and the traditional algorithm.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">dimension, equation, horizontal, partial-product method, vertical</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 156-173)
• Partial-Products Method Poster
• 1 red and1 blue colored pencil
• Whiteboard with marker</td><td>• Student Booklet (pp. 79-87)
• Whiteboard with marker (1 per student)
• Partial-Product Method Bookmark (1 per student)
• 1 red and 1 blue colored pencil (1 set of pencils per student)</td></tr>
</table>

Preview

Say: **Previously, we used an area model and the partial-products method to solve 2-digit by 1-digit multiplication problems. Today we will use the same strategy to solve 2-digit by 2-digit problems.**

Engage Prior/Informal Knowledge — Time: 3 min

Review how to estimate and solve a 2-digit by 1-digit multiplication problem.

Have students work out the problem 18 × 4 on their whiteboards. Work along with the students on your whiteboard.

Ask questions about the problem as you go, such as:

- How can you estimate the solution to this problem? *(round the factor with 2-digits to the nearest 10, then multiply by the second factor)*
- What would the area model look like for this problem? *(a rectangle that is 18 units by 4 units)*
- Which factor do you break apart into facts you can solve mentally? *(18; into tens and ones, 1 ten and 8 ones or 10 and 8)*
- How do you show breaking apart a factor with the area model? *(split the rectangle with a line to show the breaking apart of 18 into tens and ones, with 10 units on one side and 8 units on the other)*
- How do you use the area model to find partial products? *(multiply the dimensions of each rectangle in the area model to find each area, which is one of the partial products)*
- How do you use partial products to solve the original multiplication problem? *(add the partial products together)*

Give the dimensions of another array. Have students tell you the multiplication sentence for the model. Discuss the relationships between the dimensions and the multiplication sentence used to find the total.

Modeled Practice Time: 8 min

1. Estimate a product and use the partial-products method to solve.

   Have students turn to the ***Modeled Practice Sheet***. Display the ***Partial-Products Method Poster***. Have students refer to their ***Partial Product Method Bookmark***. The teacher and students will complete the steps together as the lesson progresses. Distribute a red and blue colored pencil to each student.

Say: **Look at the area model. What are the dimensions of this rectangle?** *(18 by 13)*

If I want to find the area, or the total number of squares inside the rectangle, what is the multiplication problem needed to solve it? *(18 × 13)* **Write it in the blanks at the top of the page.**

To solve this problem we will use the partial-products method. First, estimate the product of this multiplication problem.

Fill in your estimate answer on your sheet. What is your estimate? How did you find it? *(responses may vary; round 18 to the tens (20),and 13 to the tens (10); 20 × 10 is 200)*

When multiplying using the partial-products method, the second step is to break apart the factor into tens and ones. This problem has 2 2-digit numbers, so we break apart both 18 and 13.

What are the 2 parts we want to break 18 into? *(10 and 8)* **Lightly shade the bottom 8 rows of the rectangle using a red colored pencil.**

How does the area model above represent the breaking apart of 18? *(it split the rectangle by shading the bottom 8 rows red and leaving the top 10 rows white)*

Label the new vertical dimensions of the 2 smaller rectangles along the side of the area model. What is the width of the white rectangle? *(10 squares)* **What is the width of the red rectangle?** *(8 squares)*

Now, break apart the other factor, 13, into tens and ones. What are the 2 parts we want to break 13 into? *(10 and 3)*

Lightly shade the right 3 columns of the rectangle using a blue colored pencil. Go ahead and shade over the few overlapping red rows.

How does the area model above represent the breaking apart of 13? *(it split the rectangle by shading the right 3 columns blue and leaving the left 10 columns not shaded blue)*

Label the new horizontal dimensions of the 2 smaller rectangles along the top of the area model. What is the length of the left rectangle? *(10 squares)* **What is the length of the right rectangle?** *(3 squares)*

After we break apart the 2-digit numbers, what is the third step when solving using partial products? *(multiply by the other factor)*

We now have 4 multiplication facts that we can solve mentally or automatically.

Let's look at the 4 rectangles we created by breaking apart the 2 factors of the multiplication problem.

What are the dimensions of the white rectangle? *(10 by 10)* **What multiplication fact describes the area of this rectangle?** *(10 × 10)* **Write it inside the rectangle.**

What are the dimensions of the blue rectangle? *(10 by 3)* **What multiplication fact describes the area of this rectangle?** *(10 × 3)* **Write it inside the rectangle.**

What are the dimensions of the red rectangle? *(8 by 10)* **What multiplication fact describes the area of this rectangle?** *(8 × 10)* **Write it inside the rectangle.**

What are the dimensions of the purple rectangle, where the blue and red overlap? *(8 by 3)* **What multiplication fact describes the area of this rectangle?** *(8 × 3)* **Write it inside the rectangle.**

Why did I choose 10 × 10, 10 × 3, 8 × 10, and 8 × 3?
(separated the tens from the ones to provide new multiplication sentences that can be computed mentally)

Watch For	**Students may need concrete examples to visualize how the total for the area model does not change.** **For these students, provide a cutout of the area model. Students can cut the model into 4 pieces to see how breaking apart the factors does not change the original amount.**

Say: **We will multiply the dimensions of each rectangle to find the partial products.**

What is 10 × 10? *(100)* **Write it.**

What is 10 × 3? *(30)* **Write it.**

What is 8 × 10? *(80)* **Write it.**

What is 8 × 3? *(24)* **Write it.**

Just like when we used partial products to solve 2-digit by 1-digit problems, what is the fourth and final step? *(add the partial products)*

We have to add the partial products to find the total area of the area model, or the product of the original multiplication problem, 18 × 13. What do you do with the 4 partial products to solve 18 × 13? *(add the 4 partial products)*

Add 2 numbers at a time, starting with the first 2 partial products and then adding the second 2 partial products. Finally, add the 2 sums together.

What is 100 + 30? *(130)* **Write it to the side.**

What is 80 + 24? *(104)* **Write it.**

What is 130 + 104? *(234)* **Write it.**

Teacher Note
Work with students to add mentally instead of on their paper. First, ask what is in the hundreds place in both numbers and add it together. Next, ask what is in the tens place in both numbers and add it together. Finally, ask what is in the ones place in both numbers and add it together.

Point to the estimated product, 200.

Say: **Is our solution, 234, close to our estimates from earlier?** *(yes)*

2. Connect the partial-products method with the traditional algorithm.

Have students continue to use the ***Modeled Practice Sheet*** to complete the same problem using the traditional algorithm. The teacher and students will complete the steps together as the lesson progresses.

Say: **We will work the same problem using the traditional way, instead of the partial-products method. Let's see if we can find where the partial products are hidden in the traditional way.**

Look at the bottom of your sheet. 18 × 13 is written vertically, 1 number above the other. In the traditional way, we will first multiply the ones place.

What 2 numbers are in the ones place? *(8 and 3)*

What is 8 × 3? *(24)* **Did we answer this same multiplication problem above in the partial products method?** *(yes)*

Since 24 is a 2-digit number we must regroup the tens, like in addition. How many tens do we regroup? *(2)* **How many ones are in 24?** *(4)* **Write 4 ones in the answer and regroup the 2 in the tens column.**

Show each of the steps for the traditional algorithm carefully. Check that students are following along.

Say: **Next. we will multiply the 3 ones times 1 ten. What is 3 × 10?** *(30)* **Did we answer this same multiplication problem above in the partial-products method?** *(yes)*

Don't forget to add the extra 2 tens. What is 30 + 20? *(50)* **We have 5 tens. Write "5" in the tens place below the line. What is the product when we multiplied 18 × 3?** *(54)*

Watch For	Students may forget to add the 2 extra tens or add incorrectly. For example, a student may come up with 30, then add the 2 tens to get 32; or they may add the 2 tens to get 50, and then write "50" in the tens place instead of 5. Provide immediate correction if this occurs. Ask students what the value of 2 tens is. Demonstrate using base-10 materials if needed.

Say: **Next we move to the tens in 13 and multiply it to the ones in 18. What is 10 × 8?** *(80)* **Did we answer this same multiplication problem above in the partial-products method?** *(yes)*

80 has 0 ones and 8 tens. Write "0" in the ones place column and "8" in the tens column above.

The last multiplication problem is the tens times tens. What is 10 × 10? *(100)* **Did we answer this same multiplication problem above in the partial-products method?** *(yes)*

Don't forget the extra 8 tens. 10 tens plus 8 tens equals 18 tens. Write "18" below, next to the 0 in the ones column.

18 × 10 equals 180. Lastly, we must add the 2 products together. Did we add the partial products together in the partial-products method? *(yes)*

What is 54 + 180? *(234)*

Did we get the same answer as above? *(yes)*

What are some similarities to the 2 different methods? *(accept reasonable answers; both methods have 4 multiplication problems in the steps; both methods add the products together at the end)*

Practice Time: 8 min

Activity 1: Work the application problem together using the partial-products method. Have students turn to the *Practice Sheet* on page 80.

Say: **We will work the problem together as a group. Let's read the problem. Ready? Read: "Elijah had a birthday party at Go Cart Racing Track. He had 12 friends attend his party. It cost each friend $14 to race a go-cart around the track 5 times. The birthday boy was free. How much money was it for all 12 friends to race the track?"**

What is the question asking you to find? *(how much money it cost for 12 friends to race)* **Underline it.**

What is the important information in the problem? *(12 friends, $14 per friend)* **Circle it. Is there extra information?** *(yes, 5 times)* **Cross it out.**

How can we solve this problem? *(12 × 14)*

We will use the partial-products method to solve. Use the area model below to help break apart the factors.

Draw a line to break apart the rectangle vertically, going down, after the first 2 columns.

Draw a line to break apart the rectangle horizontally, going across, after the first 4 rows.

Let's label the new smaller rectangles. What are the dimensions for the top left rectangle? *(4 by 2)* **What is the multiplication equation?** *(4 × 2 = 8)* **Write it in the rectangle.**

What are the dimensions for the top right rectangle? *(4 by 10)* **What is the multiplication equation?** *(4 × 10 = 40)* **Write it in the rectangle.**

What are the dimensions for the bottom left rectangle? *(10 by 2)* **What is the multiplication equation?** *(10 × 2 = 20)* **Write it in the rectangle.**

What are the dimensions for the last rectangle? *(10 by 10)* **What is the multiplication equation?** *(10 × 10 = 100)* **Write it in the rectangle.**

Teacher Note
Students may switch the factors when identifying the dimensions and the multiplication problem for each rectangle. Either way is acceptable because of the commutative property.

Say: **What is the next step?** *(add the partial products together)*

Add 2 numbers at a time. First, what is 8 + 40? *(48)* **Next, what is 20 + 100?** *(120)*

Finally, adding the 2 sums, what is 48 + 120? *(168)*

How much money will it cost for the 12 friends to race the go-carts at the party? *($168)*

Activity 2: Have students turn to the *Practice Sheets* on pages 81 and 82. Students will complete the problems with their partner. Provide corrective feedback as needed.

Say: **Work with a math partner to complete the next set of problems. Don't forget to first estimate to check that your answer is reasonable.**

Possible questions to check for understanding:

- What is your estimated answer?

- Is your solution close to your estimation?
- How do you plan to break apart the rectangle to find 4 facts you can solve mentally?
- What are the dimensions of the new rectangles after you break the area model apart?
- What strategy did you use to solve this step (reference a multiplication or addition problem)?

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work for 5 minutes independently solving multiplication problems. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

ESTAR INTERVENTION

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 14

Using Area "Box" Models to Solve 2-Digit by 2-Digit Multiplication Problems

<table>
<tr><td>Lesson Objectives</td><td colspan="2">• The student will use an area model to reinforce the partial-products method with 2-digit by 2-digit numbers.
• The student will list the steps to the partial-products method for solving 2-digit by 2-digit multiplication problems.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">dimension, horizontal, partial-products method, vertical</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 174-193)
• Partial-Products Method poster
• 1 red and 1 blue colored pencil
• Whiteboard with marker</td><td>• Student Booklet (pp. 88-97)
• Partial Products Method bookmark (1 per student)
• 1 red and 1 blue colored pencil (1 set of pencils per student)</td></tr>
</table>

Preview

Say: **Previously, we used an area model with a grid and the partial-products method to solve 2-digit by 2-digit multiplication problems. Today we will use the same strategy to solve larger 2-digit by 2-digit problems without the grid.**

Engage Prior/Informal Knowledge Time: 3 min

Review how to estimate and solve a 2-digit by 2-digit multiplication problem using an area model.

Have students turn to the ***Engaged Practice Sheet***. Have students use their ***Partial-Products Method bookmark***. Display the ***Partial-Products Method poster***. The teacher and students will complete the answers together as the lesson progresses.

Say: **How can you estimate the solution to this problem?** *(round the factors to the nearest 10, then multiply the rounded factors)* **Write the estimate problem and the answer.**

The area model is already drawn for us. How do we use the area model and the partial-products method? *(multiply the dimensions of each rectangle in the area model to find each area, which is 1 of the partial products)*

After we estimate, what is the next step? *(break apart the factors into tens and ones)* **What factors do we break apart?** *(19 and 12)* **How do we break apart the factors into facts we can solve mentally?** *(19 into 10 and 9, 12 into 10 and 2)*

Write the dimensions of the area model rectangles below.

What is the third step? *(multiply by the other factor)* **Write the multiplication equation inside each rectangle.**

How do we use partial products to solve the original multiplication problem? *(add the partial products together)* **Add the partial product and write the answer.**

Teacher Note

If students are able to keep up with this quick pace review and show mastery of dividing an area model with a grid, then proceed.

If students are struggling with the partial-products method and show difficulty dividing an area model with a grid, then stop the lesson here. Use graph paper to draw rectangles. Using mathematical language from the lesson, having students find the area using the partial-products method.

Modeled Practice **Time: 8 min**

1. Estimate a product and use the partial-products method to solve using an area model without grid lines.

Have students turn to ***Modeled Practice Sheet #1***. Continue to use the ***Partial Products Method poster*** and ***bookmark***. The teacher and students will complete the steps together as the lesson progresses. Distribute the red and blue colored pencils to students.

Say: **Look at the area model. Even though it does not contain the squares drawn in, we can still use it to help us solve. Look at the dimensions labeling the length and width. What is the width?** *(32)* **What is the length?** *(57)*

What does the area of an area model represent? *(the product of a multiplication problem)*

What multiplication problem does this area model represent? *(32 × 57)* **Fill in the multiplication problem at the top.**

What is the first step when solving a 2-digit by 2-digit multiplication problem? *(estimate the product)*

What is your estimate? *(responses may vary; round 32 to the tens (30); round 57 to the tens (60); 30 × 60 is 1,800)* **Fill in the estimate on your sheet.**

Using the partial-products method, what do we do next after we have estimated the product? *(break apart the 2-digit numbers into tens and ones)*

Start with the first factor. How do we break apart 32? *(into tens and ones, 30 and 2)*

We don't have a grid, so we are going to estimate where to break apart the area model horizontally. The top part will be 30 and the bottom part will be 2. Will we divide down the middle or closer to 1 end? *(closer to 1 end)* **Why?** *(because 2 is a smaller amount than 30)*

Draw a horizontal line across the area model near the bottom, and then lightly shade the lower rectangle using a red colored pencil.

What is the width of the white rectangle? *(30)* **Label it. What is the width of the red rectangle?** *(2)* **Label it.**

Now, break apart the other factor, 57, into tens and ones. What are the 2 parts? *(50 and 7)*

Will the dividing line be closer to the right side or down the middle? *(closer to the right side because 7 is much less than 50)*

Draw a vertical line down the area model close to the right side, and then lightly shade the area to the right of the line using a blue colored pencil. Remember the red and blue colors will overlap in the smallest rectangle.

What is the width of the white rectangle? *(50)* **Label it. What is the width of the blue rectangle?** *(7)* **Label it.**

What is the next step when solving a multiplication problem using the partial-products method? *(multiply by the other factor)*

How does the area model represent these parts? *(they are the dimensions of the smaller rectangles)*

Look at the 4 rectangles we created by breaking apart the 2 factors of the multiplication problem.

What are the dimensions of the white rectangle? *(30 by 50)* **Write the multiplication equation in the rectangle.** *(30 × 50 = 1,500)*

What are the dimensions of the blue rectangle? *(30 by 7)* **Write the multiplication equation in the rectangle.** *(30 × 7 = 210)*

What are the dimensions of the red rectangle? *(2 by 50)* **Write the multiplication equation in the rectangle.** *(2 × 50 = 100)*

What are the dimensions of the purple rectangle, or where the blue and red overlap? *(2 by 7)* **Write the multiplication equation in the rectangle.** *(2 × 7 = 14)*

What is the final step for solving the original multiplication problem? *(add the partial products to find the total)*

Add the first 2 partial products, then add the second 2 partial products. Finally, add the 2 sums.

What is 1,500 + 210? *(1,710)*

What is 100 + 14? *(114)*

What is 1,710 + 114? *(1,824)*

Teacher Note
Work with students to add mentally or have students write out the addition on the side of their paper.

Point to the estimated product, 1,800.

Say: **Is my solution, 1,824, close to our estimate?** *(yes)*

2. Use the partial-products method to solve a word problem involving 2-digit by 2-digit numbers. Use the area model without grid lines to solve.

   Have students turn to ***Modeled Practice Sheet #2.*** The group will work together to solve the problem.

Say: **Let's read the problem together. Ready? Read: "Marcus volunteers at his local food bank. If the food bank collects 75 pounds of food every day, how much food will the food bank collect in October, which has 31 days?"**

What is the question asking you to find? *(how much food was collected in October)* **Underline it.**

What is some important information? *(75 pounds and 31 days)* **Circle it.**

What is the mathematical problem we need to write to help us answer the question? *(75 × 31)*

What is the first step in solving a 2-digit by 2-digit multiplication problem? *(estimate the answer)*

What is your estimate? *(responses may vary; round 31 to the tens (30); round 75 to the tens (80); 30 × 80 is 2,400)* **Fill in the estimate on your sheet.**

Draw a simple area model without the squares and label the dimensions given by the multiplication problem.

What are the dimensions of the rectangle we should draw? *(31 by 75)*

This is just a model, so the lengths of the sides do not need to be exact. Label the vertical side, the side going down, 31, and the horizontal side, the side going across, 75.

What is the next step in solving the multiplication problem? *(break apart the factors into tens and ones)*

How do we represent breaking apart the factors using the area model? *(break apart the dimensions on the sides of the area model into tens and ones)*

How should we break apart 31? *(30 and 1)*

Break apart the first factor, 31, by dividing the area model horizontally, or across. Estimate. Where will the line go? *(close to the bottom because 30 is much greater than 1)*

Draw a horizontal line across the area model near the bottom, and then lightly shade the lower rectangle using a red colored pencil.

Label the new dimensions of each rectangle.

How should we break apart 75? *(70 and 5)*

Estimate the line dividing the 2 parts.

Draw a vertical line down the area model close to the right side, and then shade the area to the right of the line using a blue colored pencil.

Label the new dimensions of each rectangle.

What is the third step in solving the original multiplication problem? *(multiply by the other factor)*

What is the multiplication equation to find the area of the white rectangle? *(30 × 70 = 2,100)* **Write it in the rectangle.**

What is the multiplication equation to find the area of the red rectangle? *(70 × 1 = 70)* **Write it in the rectangle.**

What is the multiplication equation to find the area of the blue rectangle? *(30 × 5 = 150)* **Write it in the rectangle.**

What is the multiplication equation to find the area of the purple rectangle? *(5 × 1 = 5)* **Write it in the rectangle.**

What is the final step in solving the original multiplication problem? *(add the partial products to find the total)*

What is 2,100 + 150? *(2,250)*

What is 70 + 5? *(75)*

What is 2,250 + 75? *(2,325)*

Teacher Note
Work with students to add mentally or have students write out the addition on the side of their paper.

Point to the estimated product, 2,400.

Say: **Does our solution, 2,325, come close to our estimate from earlier?** *(yes)*

How many pounds of food did the food bank collect during the month of October? *(2,325 pounds)*

Practice

Time: 8 min

Activity 1: Have students turn to the ***Practice Sheets*** on pages 91 and 92. Students will complete the problems with their partner. Provide corrective feedback as needed.

- What is your estimated answer?
- Is your solution close to your estimation?
- How do you plan to break apart the rectangle to find 4 facts you can solve mentally?
- What are the dimensions of the new rectangles after you break apart the area model?
- What strategy did you use to solve this step (reference a multiplication or addition problem)?

Activity 2: Have students turn to ***Practice Sheet*** on page 93. Students will use the partial-products method to solve a word problem involving 2-digit by 2-digit numbers. Students should draw an area model without grid lines to solve.

Say: **Work with a math partner to solve the problem. Draw an area model and use the partial-products method to solve.**

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes solving multiplication problems. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 15

Solving 2-Digit by 2-Digit Multiplication Problems

Lesson Objectives	• The student will solve 2-digit by 2-digit multiplication problems. • The student will follow the steps of the partial-products method.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	factor, partial-products method	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 194-205) • Whiteboard with marker • Partial-Products Method poster	• Student Booklet (pp. 98-103) • Partial-Products Method bookmark (1 per student) • Missing Step cards (5 cards per student pair)

Preview

Say: **Previously, we used an area model and the partial-products method to solve 2-digit by 2-digit multiplication problems. Today we will use the partial-products method with a multiplication square.**

Engage Prior/Informal Knowledge — Time: 3 min

Review the steps for solving a 2-digit by 1-digit multiplication problem without a model. Write "18 x 6" on the whiteboard.

Ask students questions about the problem:

- What do you do first, before solving the problem? *(estimate an answer)*
- What is the next step after we estimate? *(break apart the factors into tens and ones)*
- How do you break apart the factors into facts you can solve mentally? *(break apart 18 into 1 ten and 8 ones)*
- What is step 3, after breaking apart the 2-digit factors? *(multiply the partial products)*
- What is step 4, after you multiply? *(add the partial products to find the total)*

Modeled Practice — Time: 8 min

1. Estimate a product and use the partial-products method to solve.

   Display the ***Partial-Products Method poster***. Have students turn to ***Modeled Practice Sheet #1***. Have students use their ***Partial-Products Method bookmark*** as an aid. The teacher and student will complete the steps together as the lesson progresses.

Say: **In the previous lesson, we used an area model to help us solve each multiplication problem. For this lesson, we will think of the area model as a simple square. We will break the square into 4 parts to represent the 4 multiplication problems we must solve in order to find the product.**

What is the problem? *(24 × 48)*

This poster and your bookmarks will help us remember the steps to the partial-products method. What is the first step? *(estimate the product)*

What is your estimate? *(responses may vary; round 24 to the tens (20), round 48 to the tens (50); 20 × 50 is 1000)* **Write it.**

What is the second step to the partial-products method? *(break apart the factors into tens and ones)*

Starting with the first factor, how do we break apart 24? *(into tens and ones, 20 and 4)* **Label the top 2 parts of the square as "20" and "4."**

How do we break apart the second factor, 48? *(into tens and ones, 40 and 8)* **Label the 2 side parts of the square as "40" and "8."**

What is the next step when solving a multiplication problem using the partial-roducts method? *(multiply the parts of 1 factor by the parts of the other factor to find the partial products)*

In the previous lesson, these partial products were the dimensions of the smaller rectangles inside the area model. The parts of each product are paired to find the 4 partial products.

Watch For	Students may struggle with visualizing an area model. Provide a whiteboard to draw the area model to explain and show how the 2 products are broken into tens and ones.

Say: **Let's look at this square like a multiplication table. The numbers on the top and on the sides are the factors. We will then write the their products inside the square.**

Put your finger on the top left square. What are the 2 factors for this square? *(20 × 40)*

What is 20 × 40? *(800)* **Write it in the square.**

Put your finger on the top right square. What are the 2 factors for this square? *(4 × 40)*

What is 4 × 40? *(160)* **Write it in the square.**

Put your finger on the bottom left square. What are the 2 factors for this square? *(20 × 8)*

What is 20 × 8? *(160)* **Write it in the square.**

Put your finger on the bottom right square. What are the 2 factors for this square? *(4 × 8)*

What is 4 × 8? *(32)* **Write it in the square.**

What is the final step for solving the original multiplication problem? *(add the partial products to find the total)* **Write out the addition problems to the side.**

What is 800 + 160? *(960)* **Write it.**

What is 160 + 32? *(192)* **Write it.**

What is 960 + 192? *(1,152)* **Write it.**

Point to the estimated product, 1,000.

Say: **Is our solution, 1,152, close to our estimates from earlier?** *(yes)*

2. Use the area model to identify a mistake that happened when multiplying 2-digit by 2-digit numbers.

   Have students turn to ***Modeled Practice Sheet #2***. The teacher and students will complete the steps together as the lesson progresses.

Say: **The problem on this page already has answers. It is our job to check the work to see if a mistake was made.**

What is the problem? *(59 × 71)*

What is the first step? *(estimate the answer)*

What is your estimate? *(responses may vary; round 71 to the tens (70), round 59 to the tens (60); 70 × 60 is 4,200)*

The answer given is 1,039. Is this close to our estimation? *(no)* **Knowing that the estimate is 4,200 and the answer found is 1,039, do you think this student solved the problem correctly? Why or why not?** *(responses may vary)*

Let's use the multiplication square to try and find where the mistake happened and to find the true answer.

What is the next step in solving the multiplication problem? *(break apart the factors)*

How should we break apart 71? *(70 and 1)* **Label the top 2 parts of the square with "70" and "1."**

How should we break apart 59? *(50 and 9)* **Label the side 2 parts of the square with "50" and "9."**

What is the third step in solving the original multiplication problem? *(multiply the parts of 1 factor by the parts of the other factor)*

For each rectangle, solve the multiplication equation. Write the product inside each square. *(50 × 70 = 3500; 9 × 70 = 630; 50 × 1 = 50; 9 × 1 = 9)*

Compare the multiplication problems we have solved in our multiplication square to the multiplication problems written in the student's sample. Are any different or missing? *(5 × 70 = 350 should be 50 × 70 = 3,500)*

Correct, the student made a mistake and wrote "5" instead of "50" in the problem, 50 × 70.

Let's continue to work out the problem to find the true answer.

What is the final step in solving the original multiplication problem? *(add the partial products to find the total)*

What is 3,500 + 50? *(3,550)*

What is 630 + 9? *(639)*

What is 3,550 + 639? *(4,189)*

Point to the estimated product, 4,200.

Say: **Does our solution, 4,189, come close to our estimate from earlier?** *(yes)*

How could the student who worked the problem avoid their mistake? *(accept reasonable answers; use estimation first; draw an area model to check your work)*

Practice **Time: 8 min**

Activity 1: Have students turn to the ***Practice Sheets*** on page 100. Students will complete the problems with their partner. Provide corrective feedback as needed.

Say: **Work with your math partner to complete the multiplication problems. Use the partial-products method and the multiplication square to help organize the steps. Don't forget to estimate before finding the actual answer.**

Possible questions to check for understanding:

- What is your estimated answer?
- Is your solution close to your estimation?
- How do you remember which parts of the factors pair together?
- What strategy did you use to solve this step (reference a multiplication or addition problem)?

Activity 2: Have students play *Missing Step* using cards. With a partner, students will have a stack of 5 cards with the missing step face up on the table. The first player takes the top card. The aim is to identify the missing step in the solution and complete only that step on a whiteboard or a scratch piece of paper. Once complete, the student checks his or her work using the back of the card. If the student is correct, he or she keeps the card. The second player then picks the second card and continues the game. The winner is the player with the most cards when the time is up.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes solving multiplication problems. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 16

Solving 2-Digit by 2-Digit Multiplication Problems Using a Multiplication Square

<table>
<tr><td>Lesson Objectives</td><td colspan="2">• The student will use the partial-products method to solve 2-digit by 2-digit multiplication problems without the aid of an area model.
• The student will follow the steps of the partial-products method.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">factor, partial products method</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 206-219)
• Whiteboard with marker
• Partial-Products Method poster</td><td>• Student Booklet (pp. 104-110)
• Missing Step cards (5 cards per pair of student)
• Multiplication table (optional, 1 per student)
• Partial Products Method bookmark (1 per student)</td></tr>
</table>

Preview

Say: **Today we will use the partial-products method and the multiplication square to solve these challenging problems even faster.**

Engage Prior/Informal Knowledge Time: 3 min

Review the steps for solving a 2-digit by 2-digit multiplication problem without a model. Identify the mistake made.

Have students turn to the ***Engaged Practice Sheet***. Read over the steps together to identify the mistake.

Say: **Here is a multiplication problem done for us. Let's check to make sure it is correct.**

We will follow the steps for solving 2-digit by 2-digit multiplication problems using the partial-products method.

The first step is to estimate 28 rounds to 30 and 35 rounds to 40. 30 × 40 = 1,200. Does this look correct? *(yes)*

What is the second step? *(break apart the factors)* **How were the factors broken apart?** *(28 into 20 and 8, 35 into 30 and 5)* **Does this step look correct?** *(yes)*

What is the third step? *(multiply by the other factor)* **Does the third step look correct?** *(no)* **Why not?** *(they added instead of multiplied)*

Have students find the correct answer. If the students did not catch the mistake at this point, have them continue on.

What is the final step? *(add the partial products)* **What did they add?** *(50 + 25 = 75; 38 + 13 = 51; 75 + 51 = 126)* **Is this step correct?** *(yes, but with the wrong numbers)*

Compare the answer to the estimate. What went wrong? *(they added the partial products instead of multiplied)*

Modeled Practice **Time: 8 min**

1. Estimate a product and use the partial-products method to solve.

   Use the ***Partial-Products Method poster*** and ***bookmark*** to review the steps to the partial-products method. Have students turn to ***Modeled Practice Sheet #1***. Complete the steps to solving the problem as the lesson progresses.

Say: **What multiplication problem are we solving?** *(45 × 64)*

We will use the partial-products method along with the multiplication square to help us solve this problem.

What is the first step in solving a 2-digit by 2-digit multiplication problem? *(estimate the product)*

What is your estimate for 45 × 64? *(responses may vary; round 45 to the tens (50) and 64 to the tens (60); 50 × 60 is 3,000)* **Write it.**

When we use the partial-products method, what do we do after we have estimated the product? *(break apart the factors into tens and ones)*

What's the first factor? *(45)* **How do we break apart 45?** *(into tens and ones, 40 and 5)* **Label the top 2 parts of the square as "40" and "5."**

How do we break apart the second factor, 64? *(into tens and ones, 60 and 4)* **Label the 2 side parts of the square as "60" and "4."**

What is the next step when solving a multiplication problem using the partial-products method? *(multiply the parts of 1 factor by the parts of the other factor to find the partial products)*

Put your finger on the top left square. What are the 2 factors for this square? *(60 × 40)*

What is 60 × 40? *(2,400)* **Write it in the square.**

Put your finger on the top right square. What are the 2 factors for this square? *(60 × 5)*

What is 60 × 5? *(300)* **Write it in the square.**

Put your finger on the bottom left square. What are the 2 factors for this square? *(4 × 40)*

What is 4 × 40? *(160)* **Write it in the square.**

Put your finger on the bottom right square. What are the 2 factors for this square? *(4 × 5)*

What is 4 × 5? *(20)* **Write it in the square.**

Teacher Note

If students are not able to quickly recall the multiplication facts, a multiplication table may help to keep the lesson moving at the recommended pace. Also, additional practice and instruction of basic facts and the strategies to solve may be needed outside of the instructional time.

Say: **What is the final step for solving the original multiplication problem?** *(add the partial products to find the total)* **Write the addition problems to the side.**

What is 2,400 + 300? *(2,700)* **Write it.**

What is 160 + 20? *(180)* **Write it.**

What is 2,700 + 180? *(2,880)* **Write it.**

Point to the estimated product, 3000.

Say: **Is our solution, 2,880, close to our earlier estimate?** *(yes)*

2. Solve the application problem using the partial-products method.

Have students turn to ***Modeled Practice Sheet #2.*** The teacher and students will complete the sheet together as the lesson progresses.

Say: **Read the problem together. Ready? Read: "Raul's car can drive 28 miles on 1 gallon of gas. If he used 37 gallons of gas this month, how far did he drive?"**

What is the question asking you to find? *(how far Raul drove this month)* **Underline it.**

What is the important information? *(28 miles and 37 gallons of gas)* **Circle it.**

Continue to refer to the steps on the ***Partial Products Method poster*** and ***bookmark***.

Say: **How will we find the total number of miles Raul drove?** *(multiply 28 × 37)* **Why?** *(accept reasonable answers; we are looking for the total; the information is provided in 2 parts; it is repeating equal groups; 28 miles 37 times)* **Write the multiplication problem we need to solve.**

What is the first step when multiplying 2-digit numbers? *(estimate the answer)*

What is your estimate? *(responses may vary; round 28 to the tens (30) and 37 to the tens (40); 30 × 40 is 1,200)* **Write it.**

What is the next step in solving this multiplication problem? *(break apart the factors)* **Draw a square and divide it into 4 parts**.

How will we label the parts of the square? *(28 as "20" and "8," 37 as "30" and "7")* **Label the square.**

What is the third step in solving the original multiplication problem? *(multiply the parts of 1 factor by the parts of the other factor)*

Fill in the products for each square.

What is 30 × 20? *(600)* **Write it.**

What is 30 × 8? *(240)* **Write it.**

What is 7 × 20? *(140)* **Write it.**

What is 7 × 8? *(56)* **Write it.**

What is the final step in solving the original multiplication problem? *(add the partial products to find the total)* **Write the addition problems to the side.**

What is 600 + 240? *(840)* **Write it.**

What is 140 + 56? *(196)* **Write it.**

What is 840 + 196? *(1,036)* **Write it.**

Point to the estimated product, 1,200.

Say: **Does our solution, 1,036, come close to our earlier estimate?** *(yes)*

1,036 what? *(miles)* **How far did Raul drive this month?** *(1,036 miles)*

Practice — Time: 8 min

Activity 1: Have students turn to the *Practice Sheet* on page 107. Students will complete the problems with their partner. Provide corrective feedback as needed.

Say: **Work with your math partner to complete the multiplication problems using the partial-products method.**

Possible questions to check for understanding:

- What is your estimated answer?
- Is your solution close to your estimation?
- How did you use the multiplication square to help in finding the solution?
- What strategy did you use to solve this step (reference a multiplication or addition problem)?

Activity 2: Have students play *Missing Step* using cards. With a partner, students will have a stack of 5 cards with the missing step face up on the table. The first player takes the top card. The aim is to identify the missing step in the solution and complete only that step on a whiteboard or a scratch piece of paper. Once complete, the student checks his or her work using the back of the card. If the student is correct, he or she keeps the card. The second player then picks the second card and continues the game. The winner is the player with the most cards when the time is up.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes solving multiplication problems. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 17

Decomposing 3-Digit Numbers

Lesson Objectives	• The student will decompose 3-digit numbers and write them in both standard and expanded forms. • The student will identify values of individual digits in a number using correct mathematical language.	
Vocabulary	**decompose:** to break apart a number into expanded form or to break apart using divisibility rules; the process of starting with a whole number and breaking it down into parts; related to expanded notation (for example: 523 = 500 + 20 + 3 or 5 hundreds 2 tens 3 ones) **standard form:** a number written with 1 digit for each place (for example: 2,486) **expanded form:** a way to write a number that shows the place value of each digit (for example: 400 + 60 + 3 = 463) **base-10 form:** the number broken into groups based on the base-10 system (for example: 248 as 2 hundreds, 4 tens, and 8 ones)	
Reviewed Vocabulary	column, digit, multiple, partial-products method	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 220-233) • Base-10 materials: 6 hundreds, 15 tens, and 10 ones	• Student Booklet (pp. 111-117) • Multiplication Table (1 per student) • Whiteboard with marker (1 per student) • Base-10 materials: 7 hundreds, 5 tens, and 5 ones (1 set per student)

Preview

Say: **Today we will *decompose,* or break apart, 3-digit numbers. The skill of breaking apart a number will help us when we have to divide by larger numbers that are not automatic facts.**

Engage Prior/Informal Knowledge Time: 3 min

Review multiples and solving division problems by thinking of the missing factor in a multiplication problem.

Have students turn to the *Engaged Practice Sheet*. Distribute a *Multiplication Table* to each student.

Say: **12 is a multiple of 6. What is a multiple?** *(the product of a number and another factor)*

List the multiples of 6 on your sheet. Use the multiplication table as needed.

Now think back to when we estimated answers to division problems. Look at #2 on your sheet.

Is 17 a multiple of 4? *(no)* **Find the 2 multiples of 4 that are closest to 17.** *(16 and 20)*

Which multiple is 17 closer to, 16 or 20? *(16)* **What is 16 ÷ 4?** *(4)*

So, 17 ÷ 4 is about what? *(4)* **Write it.**

Have students complete the rest of the sheet on their own. Provide corrective feedback when necessary. Quickly review the answer before moving to the next section.

Modeled Practice Time: 8 min

1. Build 3-digit numbers using base-10 materials to show that decomposing the number does not change the value.

   Build the number 562 using 5 hundreds, 6 tens, and 2 ones. Distribute a whiteboard to each student. Have students turn to the *Modeled Practice Sheet.* The teacher and students will complete the sheet as the lesson progresses.

Say: **I built a number with *base-10* materials.**

How many groups of 100? *(5)* **Write "5" on the hundreds blank on your sheet.**

How many groups of 10? *(6)* **Write "6" on the tens blank on your sheet.**

How many ones? *(2)* **Write "2" on the ones blank on your sheet.**

This is the *base-10 form* of the number. The *base-10 form* is a number in the form of groups based on the *base*-10 system.

How many in all? *(562)* **Write it.**

562 is the *standard form* of the number. *Standard form* means to write 1 digit for each place. How many digits are in this number? *(3)*

This is just 1 way to make 562. We are going to *decompose,* or break apart, 562 in a different way.

***Decompose* means to break apart a number into different parts. Think about the number 6. We can *decompose* 6 into 2 parts—3 and 3. What is 3 + 3?** *(6)* **So, how many in all?** *(6)* **What is another way to *decompose* 6?** *(answers may vary; 1 and 5; 2 and 4; 2 and 2 and 2)* **Regardless of how we break a whole number apart, it does not change how many in all.**

1 way to *decompose* 562 is to write it in *expanded form*. *Expanded form* means to write a number by showing the value of each digit.

What is the value of 5 groups of 100? *(500)* **Write it.**

What is the value of 6 groups of 10? *(60)* **Write it.**

What is the value of 2 groups of ones? *(2)* **Write it.**

Write an addition symbol between each value to show that we can add the parts to still equal the whole, 562.

Do you think there are other ways to *decompose* or break apart 562? *(yes)* **Why?** *(answers may vary; regrouping for less hundreds or more ones; equal sharing)*

2. Build 562 another way using base-10 materials.

   Have a student volunteer exchange 1 ten for 10 ones.

Say: **We are exchanging 1 group of 10 for how many ones?** *(10)*

Does this change the value of the number? *(no)*

Write the *base-10 form* after *decomposing* 562 this way.

How many groups of 100? *(5 hundreds)* **Write it.**

How many groups of 10? *(5 tens)* **Write it.**

How many ones? *(12 ones)* **Write it.**

5 hundreds, 5 tens, and 12 ones is *base-10 form*. Does it still equal 562 in all? *(yes)* **How?** *(12 ones is 1 group of 10 and 2 ones)*

Can we write 5 hundreds, 5 tens, and 12 ones in *standard form*, 1 digit for each place? *(no)* **Why not?** *(because the ones place has more than 9)*

Five hundred fifty-twelve is not *standard form*. We have *decomposed* this number in a different way than regular *standard form*.

Let's write the *expanded form* for this *base-10 form*.

What is the value of 5 hundreds? *(500)* **Write it.**

What is the value 5 tens? *(50)* **Write it.**

What is the value 12 ones? *(12)* **Write it.**

Read the *expanded form* we just wrote. *(500 + 50 + 12)*

Can we build this number in a different way? *(yes)*

Replace the 10 ones with the 1 ten.

Say: **How could we build this number using only tens and ones? How many groups of 10 is 5 hundreds?** *(50 tens)*

How many tens do we already have in the tens place for 562? *(6 tens)* **How many tens is that altogether?** *(56 tens)*

How many ones do we have in the ones place for 562? *(2)*

What would be the *base-10 form* for 562 in only tens and ones? *(56 tens and 2 ones)*

Practice — Time: 8 min

Activity 1: Students will work to decompose numbers on their own. Then have students compare their answers with a math partner.

Have students turn to the *Practice Sheet* on page 113.

Say: **Write the number in *expanded form* then *decompose, or* break it apart, it another way.**

When you are finished, compare your answers with your partner. They may have a different way of *decomposing* the number. Check their work to see if it is still equal to the original number.

Provide students time to discuss and share their answers. If students are unsure of how to share, prompt student discussion by asking 1 of the following questions:

- How did your partner decompose the number?
- How is it different from your answer?
- Is there a place value your partner did not use in their decomposing of the number? (e.g., they did not use any tens)

Activity 2: Students work in pairs to write 3-digit numbers in standard form and decompose the number at least 2 different ways.

Distribute whiteboards and markers to each student. Read a number below. Have students write the number on their whiteboards, then work with their math partner to decompose the number in 2 different ways.

Say: **I will read you a number. Write the number on your whiteboard in *standard form*. Compare your number with your partner's number to check that you have written the same number that was read.**

Then you and your partner will work out 2 different ways to *decompose* the number. You can write it with all tens and ones, or with hundreds and ones, or another way.

When you and your partner both have a different way to write the number, place your whiteboards down so I know we are ready to share.

Possible numbers: 890, 657, 212.

Ask the following questions to check for understanding:

- What is one way to decompose this number? *(examples for 890: 8 hundreds, 8 tens, and 10 ones; 7 hundreds, 19 tens, and 0 ones)*
- What is the value of each digit? *(answer for 890: 800, 90, and 0)*
- Can you decompose this number using only tens and ones? *(examples for 890: yes, 89 tens and 0 ones, or 80 tens and 90 ones)*

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as much as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 18

Division as Equal Sharing Using Hundreds, Tens, and Ones

Lesson Objectives	• The student will solve and represent 3-digit by 1-digit division sentences in picture, word, and number forms using hundreds, tens, and ones. • The student will use mathematical terminology relating to division when discussing how to solve a division problem.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	decompose, dividend, division, divisor, equal share, quotient, remainder	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 234-253) • Base-10 materials: 2 hundreds, 25 tens, and 10 ones	• Student Booklet (pp. 118-127)

Preview

Say: **Today we will use hundreds, tens, and ones to divide dividends with 3 digits.**

Engage Prior/Informal Knowledge **Time: 3 min**

Review place value concepts by writing numbers in standard form, base-10 form, and expanded form.

Have students turn to the ***Engaged Practice Sheets***. Read the following numbers to students: 334, 187, 863, and 902.

Say: **I will read some numbers to you. Write the numbers in standard form first on your sheet. Then, we will write the numbers in base-10 form and in expanded form.**

After students have completed the questions on the sheet, ask the following questions:

- How could we decompose 334 using only tens and ones? *(33 tens and 4 ones)* 187 using only tens and ones? *(18 tens and 7 ones)*
- How many groups of 10 are in 863? *(86 tens)* How many groups of 10 are in 900? *(90 tens)*

Write the following base-10 forms on the whiteboard and ask questions such as:

- What is 7 hundreds, 16 tens, and 3 ones in standard form? *(863)*
- What is 5 hundreds, 44 tens, and 2 ones in standard form? *(942)*

Modeled Practice **Time: 8 min**

1. Using base-10 materials, model 3-digit by 1-digit division with tens and ones using the correct vocabulary to describe a division problem.

Have students turn to ***Modeled Practice Sheet #1***. With base-10 materials, build 242 with 2 hundreds, 4 tens, and 2 ones. The teacher and students will complete the steps together as the lesson progresses.

Say: **I will build a number to show, or represent, an amount of gems. We will share the gems among the bags.**

How many gems do we have in total? *(242)*

The total amount in a division problem is called the dividend. What is this total called? *(the dividend)*

Point to the 2 hundreds.

Say: **How many groups of 100 in 242?** *(2)* **Can we share 2 groups of 100 among 4 bags without breaking the hundreds apart?** *(no)*

Decompose, or break apart, the 2 groups of 100 into tens to share equally. How many groups of 10 are in 200? *(20)*

Have a student volunteer trade 2 hundreds for 20 tens. Point to the 20 tens and the 4 tens already there.

Say: **Now that we have decomposed the 2 hundreds into 20 tens, how many total groups of 10 gems do we have?** *(24)*

How many ones are in 242? *(2)*

Write "242" on the "Dividend" line. Above it, write "24" on the "Tens" line and "2" on the "Ones" line.

How many bags do we share the gems among? *(4)* **4 is the number we are dividing by. This number is called the divisor because it divides.**

Write "4" on the "Divisor" line.

We need to equally share 242 gems among 4 bags. What does it mean to share the gems equally? *(to give the same amount of gems to each person until there are none left to be shared)*

To find the equal share, we have to distribute the gems so that each bag gets the same amount.

We can equally share the gems using the tens and ones.

Point to the 24 tens.

Say: **If we give 1 group of 10 gems to 1 bag, how many groups of 10 do we give to the others?** *(1)* **Why?** *(we have to equally share all gems among all of the bags)*

Have a student volunteer continue sharing the tens among the bags, moving the tens to the bags as they are shared.

Say: **How many groups of 10 gems does each bag get?** *(6)*

Write "6" on the "Tens Equal Share" line.

Are we finished equal sharing? *(no)* **Why?** *(there are still single gems, or ones, that I can share equally)*

Point to the 2 ones.

Say: **How many gems do we have left to share?** *(2 gems)* **If we don't break them apart, can we share them equally?** *(no)* **Why?** *(we only have 2 gems and there are 4 bags)*

How many gems do we have left over? *(2)* **What is the mathematical word for left overs?** *(remainder)*

Where do we put the 2 gems that we could not share? *(in the remainder)*

How many single gems, or ones, does each bag get? *(0)*

Do we leave the ones place blank? *(no)* **What number do we write in the ones place to represent that there are no ones?** *(0)* **Write "0" on the "Ones Equal Share" line.**

How many groups of 10 gems does each bag get? *(6)*

What is the value of 6 tens? *(60)* **How many total gems does each bag get?** *(60)*

Write "60" on the "Quotient" line.

Because each bag has the same amount of gems, the gems have been shared equally. We have 2 gems that could not be shared, so 2 is our remainder.

Write "2" on the "Remainder" line.

2. Model 3-digit by 1-digit division with hundreds, tens, and ones with correct vocabulary.

   Have the students turn to ***Modeled Practice Sheet #2***. Draw the number 673 with 6 squares, 7 lines, and 3 circles. The teacher and students will complete the steps together as the lesson progresses.

Say: **We will draw a number to show, or represent, an amount of gems. We will share these gems between the bags shown here.**

Draw 6 squares. Each square represents 1 hundred. What is the value of the 6 hundreds? *(600)*

Draw 7 straight lines. Each line represents 1 ten. What is the value of the 7 tens? *(70)*

Draw 3 quick circles. Each circle represents 1 one. What is the value of the 3 ones? *(3)*

What number did we just draw? *(673)* **This number is the total we will be dividing. What is this total called in a division problem?** *(the dividend)* **Write it on the "Dividend" line.**

How many bags do we share the 673 gems among? *(3)* **What is the mathematical word for the number that tells us how many to equally share with or divide by?** *(divisor)* **Write it on the "Divisor" line.**

We need to equally share 673 gems among 3 bags. What does it mean to share the gems equally? *(to give the same amount of gems to each person until there are none left to be shared)*

Point to the picture of the 6 hundreds.

Say: **Are there enough hundreds for each bag to get 1 group of 100?** *(yes)* **On the "Hundreds" place line in the dividend, write "6." Fill in the tens and ones places in the dividend.**

If we give 1 group of 100 gems to 1 bag, how many groups of 100 do we give to the others? *(1)* **Why?** *(we have to equally share all gems among all the bags)*

Do we need to decompose the groups of 100 into groups of 10 to share equally? *(no)* **Why?** *(6 divided by 3 is 2 groups of 100 per bag)*

Draw a hundreds square inside each bag and cross it out on top to show that it has been used. Continue until all squares or hundreds are gone.

How many groups of 100 gems does each bag get? *(2)*

Write "2" on the "Hundreds Equal Share" line.

Point to the picture of the 7 tens.

Say: **We have 7 groups of 10 to share. Do we have enough tens for each bag to get 1 ten?** *(yes)* **How do you know?** *(7 is greater than 3)*

Share by drawing the tens or lines in each bag to see how many groups of 10 each bag will receive.

How many groups of 10 gems does each bag get? *(2)* **Write "2" on the "Tens Equal Share" line.**

Are there any groups of 10 left? *(yes)*

How many groups of 10 are left? *(1)*

Can we give this group of 10 to just 1 bag? *(no)* **Why?** *(not equal sharing)*

What do we have to do with this groups of 10 to share equally? *(break it into ones)*

We can share the 1 group of 10 by decomposing or breaking it apart into ones. How many ones in 1 group of 10? *(10)*

Cross out 1 group of 10 and draw 10 circles.

How many total ones do we have? *(13)* **We can continue sharing the gems. How do we know when we are finished sharing?** *(when no gems are left to share or when I can't give the same amount of gems to each bag)*

Do we have any gems leftover? *(yes)* **How many are left over?** *(1)* **Where do we put this 1 gem that we could not share?** *(in the remainder)*

How many single gems, or ones, does each bag get? *(4)*

Write "4" on the "Ones Equal Share" line.

How many groups of 100 gems does each bag get? *(2)*

How many groups of 10 gems does each bag get? *(2)*

How many groups of 1 gem does each bag get? *(4)*

How many total gems does each bag get? *(224)*

Write "224" on the "Quotient" line. What is another word for quotient? *(answer)*

Each bag has the same amount of gems, therefore the gems have been shared equally. We have 1 gem that could not be shared. Where do we record this? *(in the remainder)*

Write "1" on the "Remainder" line.

3. Review the division problems and solutions.

Have students turn back to *Modeled Practice Sheet #1*.

Say: **To solve this division problem, what did we have to do with the hundreds?** *(decompose; break them apart; trade them for tens)*

Why? *(there were not enough hundreds for every bag)*

Have students turn back to *Modeled Practice Sheet #2.*

Say: **To solve this division problem what happened with the hundreds?** *(they were divided equally)*

Why did we not trade them for tens? *(because there were enough for each bag to get 2)*

Practice Time: 8 min

Activity 1: Students will complete the *Practice Sheets* on pages 122 and 123 as a group. Have students use the base-10 pictures to solve.

Say: **We will work these division problems together.**

Use the base-10 pictures to solve.

As students are completing the problems, have them verbalize their reasoning by asking questions such as:

- Are we able to equally share the hundreds? Why?
- Are we able to equally share the tens?
- Do you need to decompose or trade any of the hundreds or tens? Why not?
- How do you know when you have shared the gems equally?
- What is the remainder?

Activity 2: Complete an application problem using base-10 drawing to help solve. Have the students turn to the *Practice Sheet* on page 124.

Say: **Let's read the problem together. Ready? Read: "Peter was helping out at his uncle's store. He was given 4 piñatas and 895 pieces of candy and prizes. The piñatas cost $24 each. Peter's uncle told him to fill each piñata with the same about of candy and prizes. How many pieces of candy and prizes will Peter put in each piñata?"**

What is the question asking you to find? *(the number of candy and prizes in each piñata)* **Underline it.**

What is the important information in the story? *(4 piñatas, 895 pieces of candy and prizes)* **Circle it.**

What mathematical operation do we need to use to solve this problem? *(division)* **How do you know?** *(the story says he has to fill each with the same amount; the story gives a whole and 1 part and asks what is the other part)*

Write the division problem for this story. *(895 ÷ 4)*

Draw base-10 pictures to solve. How many squares for hundreds will you draw? *(8)*

How many lines for tens? *(9)*

How many quick circles for ones? *(5)*

Draw 4 boxes to represent the 4 piñatas. Divide and show your work using the base-10 picture.

Check students' work. Have student verbalize how they solved the problem.

Say: **How many candies and prizes did Peter put in each piñata?** *(223)*

Are there any pieces remaining? *(yes)*

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes solving division problems. Complete as many as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

ESTAR INTERVENTION

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 19

3-Digit Division Using Hundreds, Tens, and Ones

<table>
<tr><td>Lesson Objective</td><td colspan="2">• The student will solve 3-digit by 1-digit division problems using hundreds, tens, and ones.
• The student will name the dividend, divisor, and quotient in a division problem.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">dividend, division, divisor, efficient, equal share, quotient</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 254-267)
• Whiteboard with marker</td><td>• Student Booklet (pp. 128-134)
• Whiteboard with marker (1 per student)</td></tr>
</table>

Preview

Say: **Today we will continue to divide 3-digit numbers by 1-digit numbers. With each problem, you will have to decide if it is better to write the dividend as tens and ones, or as hundreds, tens, and ones to share equally.**

Engage Prior/Informal Knowledge — Time: 3 min

Review place value and sharing using tens and ones. Distribute whiteboards to each student. Have the students write "334" on the whiteboard.

Say: **Write the number "334" on your whiteboard. We will decompose the number in several different ways.**

Prompt with the following questions and instructions:

- Write 334 as hundreds, tens, and ones in base-10 form. *(334 is 3 hundreds, 3 tens, and 4 ones)*
- Suppose we were sharing 334 erasers among 3 people. Does it make sense to write it as groups of hundreds, tens, and ones? *(yes)* Why or why not? *(3 hundreds can be shared among 3 people, as can 3 tens, etc.)*
- What if we were sharing 334 among 4 people? Does it still make sense to write it as groups of hundreds, tens, and ones? *(no)*
- How would you have to write 334 in order to share equally among 4 people? *(as tens and ones)* How many tens? *(33 tens)* How many ones? *(4 ones)*

Modeled Practice

Time: 8 min

1. Use base-10 pictures to solve 3-digit by 1-digit division with tens and ones using the correct division vocabulary.

   Have students turn to ***Modeled Practice Sheet #1.*** The teacher and students will complete the steps together as the lesson progresses.

Say: **The base-10 picture represents erasers.**

How many erasers do we have in total? *(453)* **What is this total called?** *(the dividend)* **Write "453" on the "Dividend" line.**

How many bags do we share the erasers among? *(3)* **What is the number doing the dividing called?** *(the divisor)* **Write "3" on the "Divisor" line.**

Point to the 4 hundreds.

Say: **How many groups of 100 are in 453?** *(4)* **Can we share 4 groups of 100 among 3 bags without breaking the hundreds apart?** *(yes)*

Because we do not need to decompose hundreds into tens, we can write the number as hundreds, tens, and ones.

How many groups of 100 erasers are in 453? *(4)*

How many groups of 10 erasers are in 453? *(5)* **How many ones are in 453?** *(3)*

Write "4" on the "Hundreds" line, "5" on the "Tens" line, and "3" on the "Ones" line.

We need to divide 453 erasers among 3 bags. How do we divide 453? *(starting with the hundreds, give the same amount to each bag)*

Share the hundreds among the bags, crossing out each hundred at the top as you draw it in each bag.

How many groups of 100 erasers does each bag get? *(1)* **What is the value of 1 hundred?** *(100)* **So, how many erasers does each bag have right now?** *(100)*

Write "1" on the "Hundreds Equal Share" line.

We have 1 group of 100 left to share. If we don't break it apart, can we share it equally? *(no)* **Why?** *(you only have 1 group of 100 erasers and there are 3 bags)*

What do we have to do with this group of 100 to share equally? *(break it into tens)*

How many tens are in 1 group of 100? *(10)*

Cross out 1 hundreds square and draw 10 lines to represent the 10 tens.

How many total groups of 10 do we have now? *(15)*

Continue sharing the groups of 10 erasers. Cross out each ten and draw it in each bag.

How do we know when we are finished dividing the tens? *(when no groups of 10 erasers are left to share or when I can't put the same amount of erasers in each bag)*

How many groups of 10 does each bag get? *(5)* **What is the value of 5 tens?** *(50)* **So, how many erasers does each bag have right now?** *(150)*

Write "5" on the "Tens Equal Share" line.

We have no groups of 10 left over. Are we finished dividing? *(no)* **Why not?** *(we still have ones to divide)*

Share the ones with the bags. How many single erasers, or ones, does each bag get? *(1)* **Write "1" on the "Ones Equal Share" line.**

Are there any single erasers remaining that I could not share? *(no)*

How many total erasers does each bag get? *(151)* **What is the answer to a division problem called?** *(quotient)*

Write "151" on the "Quotient" line.

Did we end up with a remainder? *(no)* **What should we write on the remainder line?** *(0)* **Write "0" on the "Remainder" line.**

2. Model a non-example. Correct the mistake using base-10 pictures.

   Have students turn to ***Modeled Practice Sheet #2*** showing 327 erasers and 2 bags.

Say: **Read the problem together. Ready? Read: "Donovan was given the division problem 327 ÷ 2. He decided to draw a base-10 picture to help solve the problem. Is this the most efficient way to solve this problem?"**

Look below at Donovan's picture. What did he draw? *(32 lines for tens and 7 circles for ones)*

What is the question asking you to find? *(if this is the most efficient way to solve this problem)*

What does efficient mean? *(the quickest way with the least amount of steps)*

What is the divisor in this problem? *(2)* **Why?** *(because we are dividing it into 2 bags)* **Write "2" on the "Divisor" line.**

How many groups of 100 are in 327? *(3)*

Could Donovan share any of the 3 groups of 100 between 2 bags without breaking the hundreds apart? *(yes)*

Draw a picture for how you would solve 327 ÷ 2.

How many groups of 100 will you draw? *(3)* **How many tens?** *(2)* **How many groups of ones?** *(7)*

Write "3" on the "Hundreds" line, "2" on the "Tens" line, and "7" on the "Ones" line.

Share the hundreds between the bags, crossing out each hundred as you draw it in each bag.

How many groups of 100 erasers does each bag get? *(1)*

Write "1" on the "Hundreds Equal Share" line.

We have 1 group of 100 left to share. What do we have to do with this group of 100 to share equally? *(break it into tens)*

How many tens in 1 group of 100? *(10)*

Cross out the hundreds square and draw 10 lines to represent the 10 tens.

How many total groups of 10 do I have? *(12)*

Divide the groups of 10 erasers, crossing out the tens lines as you draw them in the bags.

How many groups of 10 does each bag get? *(6)* **Write "6" on the "Tens Equal Share" line.**

How do we know when we are finished sharing the tens? *(when no groups of 10 erasers are left to share or when I can't put the same amount of erasers in each bag)*

We have divided the hundreds and tens. What is next? *(divide the ones)*

Divide the groups of 1 eraser, crossing out the ones circles as you draw them in the bags.

How many single erasers, or ones, does each bag get? *(3)* **Write "3" on the "Ones Equal Share" line.**

Are there any ones erasers left over that we could not share? *(yes)* **What is the remainder?** *(1)* **Write "1" on the "Remainder" line.**

Point to the "Quotient" line.

Say: **We have 1 group of 100, 6 groups of 10, and 3 ones. What is the quotient?** *(163)* **Write "163" on the "Quotient" line.**

Each bag has the same amount of erasers, so the erasers have been shared equally.

Would Donovan get the same answer by breaking all the hundreds into tens? *(yes)* **So what is wrong with doing it the way Donovan set it up?** *(it is not efficient; it would take much longer)*

Practice — Time: 8 min

Activity 1: Students will work with a math partner to complete the division problems using base-10 pictures.

Have students turn to the *Practice Sheet* on page 130.

Say: **With your math partner, complete the division problems. Use base-10 pictures to solve.**

Have a few students verbalize their reasoning, asking the following questions:

- How many hundreds, tens, and ones did you draw?
- What is the approach (steps) to share the erasers equally?
- How do you know when you have shared the erasers equally?
- What do we do with the erasers we could not share?
- What is the dividend? The divisor? The quotient?

Activity 2: Students will complete the ***Practice Sheet*** on page 131 as a group.

Say: **Let's solve the problem together. Ready? Read: "There were a total of 495 fans at the 3 play-off games. If the same number of fans attended each game, how many fans attended the first game?"**

As students complete the division problem, ask questions such as:

- What is the question asking you to find? *(number of fans at a game)*
- What mathematical operation will you use to solve this problem? *(division)* Why? *(accept reasonable answers; the story gave us the total and 1 part and asked for the other part)*
- How many hundreds, tens, and ones will you draw? *(4 hundreds, 9 tens, and 5 ones)*
- What is the dividend? *(495)* The divisor? *(3)*
- Can you divide 4 groups of 100 into 3 groups without breaking the hundreds into groups of 10? *(yes)*
- What is the hundreds equal share in the quotient? *(1)*
- How many total groups of 10 do you have now after you divided the hundreds? *(19)*
- How do you know when you are finished dividing? *(when no groups of 10 are left to share or when I can't give the same amount to each group)*
- How many fans attended each game? *(165 fans)*

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as much as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division of Whole Numbers
Lesson 20

Multiplication and Division: Application Problems

<table>
<tr><td rowspan="1">Lesson Objectives</td><td colspan="2">• The student will estimate to find reasonable answers for multiplication and division application problems.
• The student will determine whether an application problem requires multiplication or division to solve based on the mathematical language of the problem.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">division, estimate, multiplication, reasonable</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 268-279)</td><td>• Student Booklet (pp. 135-140)
• Multiplication table (1 per student)
• Partial-Products Method bookmark (1 per student)</td></tr>
</table>

Preview

Say: **Today we will be finding reasonable answers to application problems. Some problems will require multiplication while other problems will require division to solve.**

Engage Prior/Informal Knowledge **Time: 3 min**

Discuss estimation in application problems. Have students brainstorm words and phrases that appear in application problems that imply the actual answer does not have to be found.

Examples: "Estimate the answer…," "About how many…," "What would be a reasonable amount…," "Approximately…"

Discuss estimation and review how to estimate multiplication and division answers. Have students turn to the ***Engaged Practice Sheet***. Work the problems together.

Ask questions and give instructions such as:

- What does 39 round to? *(40)*
- How would you estimate 51 x 59? *(50 times 60)*
- Rewrite "55 ÷ 9" as a multiplication problem with a missing factor. *(n x 9 = 55)*
- Listing the multiples of 9 to get close to 55, do we need to start at 9? *(no, we can start at 27 or 36 or 45)*
- Estimating 321 ÷ 7, what 2 digits in 321 can we look at to get a good estimation? *(32)*

Modeled Practice **Time: 8 min**

1. Work through application problems using 4 steps to solve.

   Have students turn to the ***Modeled Practice Sheet***. Read the problem at the top together or have a student volunteer read it aloud. The teacher and students will complete the steps together as the lesson progresses.

Say: **The first step in solving a problem is to determine what the question is asking you to find.**

Reread the problem to yourself then tell me what the question is asking you to find. *(the number of 2-point shots the team scored)*

Fill in your answer in the first box under the question.

We have identified the unknown. Now we need to find the important information from the problem to answer the question.

What is the important information? *(103 points and 2-point shots)* **Circle it.**

What does 103 points represent in the problem? *(the total number of points earned in the game)*

How much is each shot worth? *(2 points and some 1 point)*

Do we know how many 2-point shots were made? *(no)*

The problem stated the total and 1 part. We are looking for the number of shots, or the missing part of the total.

What operation or method of solving should we use when we know the total and 1 part but need to find a missing part? *(methods may vary; accept methods that are mathematically correct and efficient, such as division or multiplication with a missing factor)*

Teacher Note

Students may use different methods to solve and each method may lead to the correct answer. Allow a few students to share their ideas, then pick 1 method for solving at this time without disregarding other students' methods. Acknowledge that there is more than 1 way to solve a problem.

Say: **Write it in the second box.**

Does the problem want to know exactly how many 2-point shots were made or an estimate? *(estimate)* **How do you know?** *(the questions says, "Estimate about how many…")*

The third step to solving a problem is to work it out and show your method through your work. For this problem we do not have to do the exact work, just estimate.

How would we show the work to go with the method we choose? *(answers may vary depending on method being used; with any method, make sure the strategy and work are evident in order to find the estimate or answer; e.g., students may write a division problem or a multiplication with a missing factor)*

Use mathematical language from previous lessons.

Possible questions to ask while working out the method that was chosen:

- Are we looking for the actual answer or an estimation?
- What are the strategy steps we follow in order to divide/multiply/find the partial product, etc.…?
- What will we break apart?
- Which number is the dividend for this division problem?
- Which number is the product for this missing factor multiplication problem?

Say: **About how many 2-point shots did the team make?** *(50 or 51)*

The last step in solving any problem is to ask yourself: Does your answer make sense?

We stated that the team scored 50 (or 51) 2-point shots during the game. Let's ask ourselves some checking questions. Is that possible in basketball? *(yes)*

Does 50 shots times 2 points each equal close to 103 points? *(yes)*

Did you answer the question: estimate how many 2-point shots the team made during the game? *(yes)*

Does our answer make sense? *(yes)* **Write your reason for why your answer makes sense. Something like, "50 + 50 = 100, which is close to 103."**

2. Answer the second application problem with the group.

   Point to problem #2 on the ***Modeled Practice Sheet***. Read the problem together or have a student volunteer read the problem aloud. Use mathematical language and think aloud as you solve the problem using the 4 steps to solve.

Say: **What is the first step in our 4 step problem-solving method?** *(state what the question is asking you to find)*

Reread the problem to yourself and tell me what the question is asking you to find. *(the total points David scored from 3-point shots)* **Fill in your answer in the first box under the question.**

What is the important information in the problem? *(17 3-point shots)* **Circle it.**

What is the second step, or question we should ask ourselves when solving a problem? *(which method you will use to solve)*

Before choosing a method to solve we must think about the information the problem provided.

What does the 17 in the problem represent? *(the number of 3-point shots he made)*

How many points is each of those shots worth? *(3 points)*

To build a mathematical equation, we should have 2 parts and 1 whole or total. Do we have the parts or the whole? *(2 parts)*

Do we need to solve to find the missing part or the missing total? *(the missing total)*

When we know the parts and are looking for the total, which method should we use to solve? *(multiplication)* **Write the multiplication problem in the second box.** *(17 × 3)*

For step 3, what strategy should you use to solve this multiplication problem? *(partial-products method)*

Before solving, let's estimate so we can use this answer to check our final answer.

What would be a good estimation for 17 × 3? *(20 × 3 = 60 or 15 × 3 = 45)*

What strategy steps should we use to solve? *(the partial-product method, 10 × 3 + 7 × 3)*

Work out the steps using the partial-products method together. Use mathematical language from previously taught lessons. Show all the steps that are used in the strategy steps to solve.

Say: **What does 17 × 3 equal?** *(51)*

Finally, before we are finished with this problem we must ask ourselves the last question of the 4 steps. Does your answer make sense?

Was our solution close to our estimation? *(yes)*

What does 51 represent in the problem? *(the number of points David scored from his 3-point shots)*

Is this reasonable? Could 17 × 3 = 51? Is it possible to score 51 points in a basketball game? *(yes)*

Did we find what the question was asking? *(yes)*

Were we looking for an actual answer or an estimated answer? *(actual answer)*

Practice **Time: 8 min**

Activity 1: Students work with math partners to solve word problems involving multiplication or division. Have students refer to their ***Partial Products Method bookmark*** if needed. Have ***multiplication tables*** available for students who struggle with multiplication facts.

Say: **Work with your math partner to answer the first 2 problems. Use the 4 steps to solve the problems. Be prepared to discuss your answers.**

Have students turn to the ***Practice Sheet*** on page 137. While students work ask questions such as:

- What do you and your partner think the question is asking you to find?
- What was your reasoning for choosing that method?
- What strategy steps are you following to find the solution?
- How do you know your answer is reasonable?
- Did you use estimation in any part of your solution?

Activity 2: Students will complete the word problems on their own then interview a partner about their solution.

Have students turn to the ***Practice Sheets*** on pages 138 and 139. Students complete problems 3 and 4 on the ***Practice Sheet*** on page 138. Once students have found a solution, students will interview their math partner using the questions found on the ***Practice Sheet*** on page 139.

Interview questions:

1. What do you think the question is asking you to find?
2. Which method did you use to solve? Why?
3. What are the strategy steps you followed?
4. How did you estimate to check that your answer was reasonable?
5. Do you think you answered the original question? Explain why you think so.

Once students have completed the interview of problem 3, have the students switch roles—the interviewer will become the interviewee and answer the same interview question for problem 4.

Independent Practice **Time: 6 min**

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.